Indian National Physics Olympiad
Theory Problems and Solutions
(2006 - 2009)

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First Edition, 2009

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Published by
Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd,
Mumbai - 400088, India.

Printed at
Print Vision, Mumbai

This book can be purchased from,
In-Charge,
Publication Section,
Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd,
Mumbai - 400088, India.
Phone: 91-22-25580036, 25554712, 25567711
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Foreword

The Physics Olympiad activity has been attracting an ever growing number of students and teachers since it was launched in 1998. A key component of this activity is the Indian National Physics Olympiad Examination, popularly known as INPhO, which is conducted by the Homi Bhabha Centre for Science Education around January end each year. To meet the demands of the students and teachers a volume detailing INPhO problems from 1998-2005 was brought out in 2008. The present volume which covers the period from 2006-2009 maybe viewed as a continuation of the same – but with a difference. This time we have included the **solutions** also, and, in multiple formats: Brief Solutions and Detailed Solutions. Some of the problems have been slightly modified or corrected in order to read better. The INPhO has been a four hour exam. However, we want the reader to work on this book at a leisurely pace. Credit (marks) has been mentioned in order to give an idea of the relative difficulty level of each problem.

We urge the readers to attempt the problems and then look at the two sets of Solutions: Brief and Detailed. In this connection we want to share with the readers two stories about seminal discoveries in theoretical physics. Both of them were"problem solving" exercises and fetched the discoverers the Nobel Prize.

On Sunday October 7 1900, Max Planck was confronted with an experimental black body radiation curve in the high wavelength regime. It was a partial curve and it deviated from Wien's theoretical prediction. Planck undertook an exercise in which he attempted to reconcile Wien's law at low wavelength with the deviant experimentally observed behaviour at high wavelength. By evening he was able to fit the two pieces of this puzzle and come up with an interpolation formula. This heralded the birth of quantum mechanics and fetched Planck the Noble Prize in 1918.

In 1970 Kenneth Wilson of Cornell University was asked to give a friendly seminar on a research paper published jointly by two Italian physicists Di Castro and Jona-Lasinio. In other words his colleagues asked him to explain the work, something that we might ask our friends to do, say over a cup of tea. The deadline for the seminar was approaching and in a desperate bid to arrive at the final conclusions of the paper, Wilson invented his "own way", different from any other. This "own way" was the beautiful "renormalization group" approach which won him the Nobel Prize in 1982. In a similar fashion we once again urge the reader to attempt the problems first and develop their "own way" and not to simply look at the solutions.

In addition the book is peppered with additional comments and minor derivations. This adds value to the collection. It includes personal descriptions by Planck and Wilson about their above - mentioned discoveries. And a delightful perspective by Isidor Issac Rabi, the discoverer of nuclear magnetic resonance, on how not to approach a physics problem! Along with the historical remarks this aesthetic exercise raises the book from the level of being merely a "problem book" on physics.

We also invite the readers to write to us pointing out errors and alternate solutions. Last, but certainly not the least, we would like to thank Ms. Sarita Yadav for discussions and excellent technical assistance.

Dated:
July 1, 2009

Prof. (Dr.) Vijay A. Singh
National Co-ordinator, Science Olympiads
Homi Bhabha Centre for Science Education (TIFR)

Table of Constants

Acceleration due to gravity on Earth	g	9.80665 m·s^{-2}
Atmospheric pressure	P_{atm}	1.01325×10^{5} Pa
Atomic mass unit	$1\ u$	931.49403 MeV·c^{-2}
Avogadro number	N_A	6.02214×10^{23} mol^{-1}
Boltzman constant	k	1.38065×10^{-23} J·K^{-1}
Distance between Sun and Earth	1 A.U.	1.49600×10^{11} m
Binding energy of hydrogen atom	-	13.6058 eV
Magnitude of electron charge	e	1.60218×10^{-19} C
Mass of the Earth	M_E	5.97420×10^{24} kg
Mass of the electron	m_e	9.10938×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Mass of the Sun	$M_\odot$	1.98892×10^{30} kg
Permeability of free space	μ_0	1.2566×10^{-6} H·m^{-1}
Permittivity of free space	ϵ_0	8.85420×10^{-12} F·m^{-1}
Planck's constant	h	6.62607×10^{-34} J·s
Radius of the Earth	R_E	6.37814×10^{6} m
Radius of the Sun	$R_\odot$	6.95500×10^{8} m
Speed of Sound in air (at room temperature)	c_s	340.29 m·s^{-1}
Speed of light in vacuum	c	2.99793×10^{8} m·s^{-1}
Stefan-Boltzmann constant	σ	5.67040×10^{-8} W·m^{-2}·K^{-4}
Surface Tension of water at 20^0 C	-	7.286×10^{-2} N·m^{-1}
Universal constant of Gravitation	G	6.67428×10^{-11} N·m^{2}·kg^{-2}
Universal gas constant	R	8.31447 J·mol^{-1}·K^{-1}
Wien's constant	-	2.89777×10^{-3} m·K

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Chapter I

Problems

1 INPhO-2006

Indian National Physics Olympiad - 2006

INPhO-2006 — Jan. 29, 2006

Maximum Marks: **90**

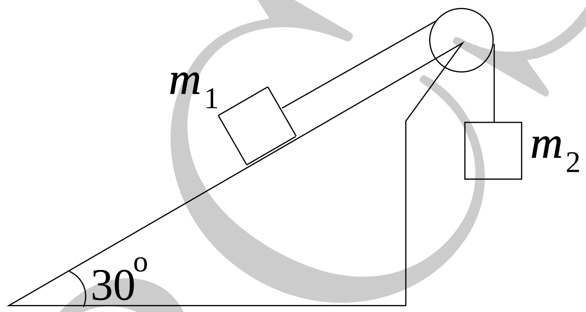

Figure 1: Problem 1

1. In the diagram shown (Fig. (1)), $m_1 = 1$ kg, $m_2 = 1$ kg and coefficient of friction, both static and dynamic, between m_1 and plane is $\mu = 0.6$. The two masses are connected by a light inextensible string passing over a light frictionless pulley. Take $g = 10$ m·s^{-2}.

   (a) Find the acceleration of the system.
   (b) Find the force of friction and the magnitude of the tension in the string.

[10]

2. A block of uniform mass M is at rest on a table. A disk of mass $2M$, radius R and of the same height as the block, which is initially spinning about its axis with angular speed ω_0, is placed on the table such that it touches the block (see Fig. (2)). The block – disk system starts moving such that they are in contact throughout the motion. Coefficient of friction, both kinetic and static, between the table and block and between the table and disk is μ. Friction between disk and the block may be ignored.

   (a) Obtain an expression for the initial acceleration of the block – disk system.

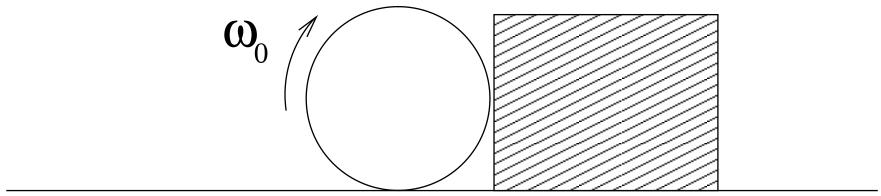

Figure 2: Problem 2

(b) Obtain an expression for the time t^* at which pure rolling (i.e. rolling without slipping) starts.

(c) Obtain an expression for the total time t_{tot} in which the block comes to the rest. Assume that pure rolling persists for $t > t^*$.

[14]

3. An ideal gas goes through a reversible cycle which consists of two isobaric and two adiabatic processes as shown in the $P-V$ diagram (Fig. (3)).

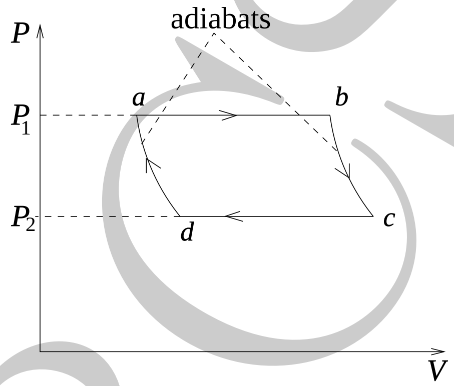

Figure 3: Problem 3

(a) Obtain an expression for the efficiency of the cycle in terms of the temperatures $\{T_a, T_b, T_c, T_d\}$.

(b) Obtain an expression for the efficiency of the cycle in terms of the pressures $\{P_1, P_2\}$ and γ. Here γ is the ratio of the specific heat at constant pressure and specific heat at constant volume.

(c) Draw the equivalent $V-T$ diagram for this cycle.
[Note: V along y-axis and T along x-axis.]

(d) State the expression for the corresponding Carnot cycle working with the same gas and between the highest and lowest temperatures defined by the above cycle. Which of these two cycles has the higher efficiency?

[12]

4. A thin plano-convex lens of radius $R = 10$ cm, refractive index $\mu_2 = 1.5$ has its curved surface in liquid of refractive index $\mu_3 = 1.2$ and the plane surface exposed to air of refractive index $\mu_1 = 1.0$. A self luminous particle oscillating simple harmonically with small amplitude $\sqrt{2}$ cm is placed on the axis of the lens as shown in Fig. (4). Determine the orientation, amplitude and phase

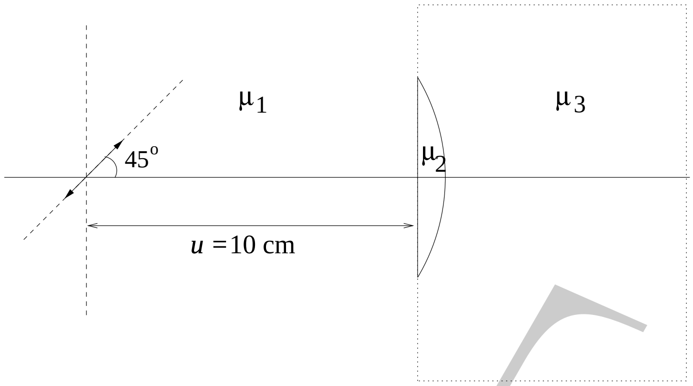

Figure 4: Problem 4

difference of the oscillating final image with respect to the object.

[8]

5. A thin circular disk of radius R is uniformly charged with charge σ ($\sigma > 0$) per unit area. The disk rotates about its axis OX with a uniform angular speed ω (see Fig. (5)). A small magnetic dipole of moment $\vec{\mu}$ is located at $P(a, 0, 0)$ on the axis of the disk ($a > 0$).

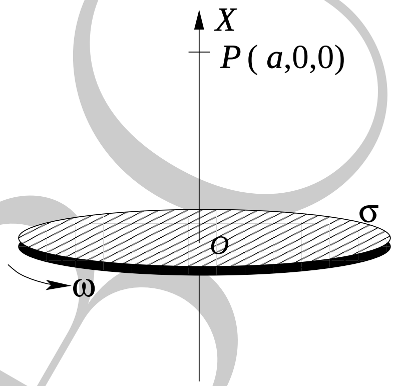

Figure 5: Problem 5

(a) Obtain the expression for the magnetic moment of the disk?
(b) Obtain the expression for the magnetic field $\vec{B}$ due to the rotating disk at P?
(c) Obtain the approximate expression for $\vec{B}$ when $a >> R$.
(d) Obtain the force on the dipole placed at P given that $a >> R$.

[Note: You can use the formula for $\vec{B}$ on the axis of a circular current, namely

$$|\vec{B}| = \frac{\mu_0}{4\pi}\ \frac{2i\pi r^2}{(r^2 + x^2)^{3/2}}$$]

[14]

6. A 1.00 kW cylindrical (monochromatic) laser light beam of radius δ is used to levitate a solid aluminium sphere of radius R by focusing it on the sphere

from below (see Fig. (6)). The laser light is reflected by the aluminium sphere without any absorption.

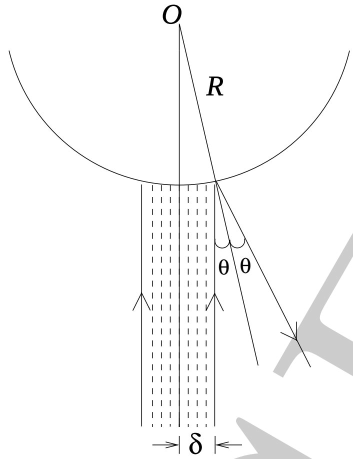

Figure 6: Problem 6

(a) Take the momentum of each photon in the light beam to be p. Express the force exerted on the aluminium sphere by the beam in terms of p, δ, R, and n where n is the number of photons per unit area per unit time.

(b) Now consider the special case $\delta << R$. Calculate the mass of the sphere, assuming that it floats freely on the light beam?

[Hint: Part (b) can be done independently of Part (a)] **[10]**

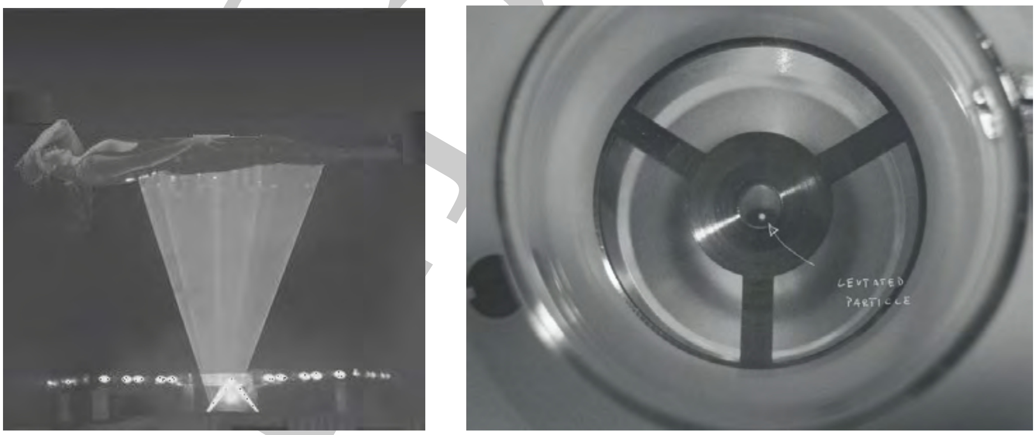

Figure 7: Demonstrations of laser levitation

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7. An electron in the Li^{++} ion makes a transition from $n = 4$ to $n = 3$ state.

(a) Find the wavelength of emitted photon in this process. To what region of the electro-magnetic spectrum does this wavelength belong?

(b) This photon impinges on a photoelectric sensitive metal having work function 3.20 eV. Calculate the maximum kinetic energy and the corresponding de Broglie wavelength of emitted photo-electron.

(Ionization energy of hydrogen atom = 13.60 eV) **[8]**

8. **Lyttleton - Bondi Model for the Expansion of the Universe***
In 1959 Lyttleton and Bondi suggested that the expansion of the Universe could be explained on the basis of Newtonian mechanics if matter carried a net electric charge. Imagine a spherical volume of astronomical size and radius R containing un-ionized atomic hydrogen gas of uniform density $\eta(= 10^{-26}\text{kg}\cdot\text{m}^{-3})$, and assume that the proton charge $e_p = -(1+y)e$, where e is the electron charge.

(a) Obtain the value of y for which the electrostatic repulsion becomes larger than the gravitational attraction and the gas expands.
(b) Obtain an expression for the force of repulsion on an atom which is at a distance R from the centre of the spherical volume. Hence show that the radial velocity is proportional to R. Let us label the proportionality constant as H. Assume that the density is maintained constant by the continuous creation of matter in space. Assume also that the value of y is larger than the equilibrium value calculated in part (a) above and hence ignore gravity.
(c) Calculate the numerical value of H. Take the value of y to be one order of magnitude larger than the equilibrium value calculated in part (a) above
(d) Given that at time $t = 0$, the volume of the Universe was V_0, obtain an expression for the volume expansion of the Universe.
(e) Why do you think the Lyttleton - Bondi model has been largely discarded by the scientific community?

[14]

*Ref. R.A.Lyttleton and H.Bondi, Proceedings of Royal Society of London, Volume **A 252**, page 313 - 333, (1959)

Raymond Arthur Lyttleton (7 May 1911-16 May 1995) : English mathematician and theoretical astronomer who researched stellar evolution and composition. In 1939, with Fred Hoyle, he demonstrated the large scale existence of interstellar hydrogen, refuting the existing belief that space was devoid of interstellar gas. Together, in the early 1940's, they applied nuclear physics to explain how energy is generated by stars. In his own monograph (1953) Lyttleton described stability of rotating liquid masses, which he extended later to explain that the Earth had a liquid core resulting from a phase change associated with a combination of intense pressure and temperature. With Hermann Bondi, in 1959, he proposed the electrostatic theory of the expanding universe. He authored various astronomy books. One of them "Mysteries of the Solar System", was co-authored with Edwin Land and was quite popular.

Sir Hermann Bondi (1 Nov.1919-10 Sept.2005) : Austrian-born British mathematician and cosmologist who, with Fred Hoyle and Thomas Gold, formulated the steady-state theory of the universe (1948). Their theory addressed a crucial problem: "How do the stars continually recede without disappearing altogether?" Their explanation was that the universe is ever-expanding, without a beginning and without an end. Further, they said, since the universe must be expanding, new matter must be continually created in order to keep the density constant, by the interchange of matter and energy. The theory was eclipsed in 1965, when Arno Penzias and Robert Wilson discovered a radiation background in microwaves giving convincing support to the "big bang" theory of creation which is now accepted.

"Sometimes I am a little unkind to all my many friends in education ... by saying that from the time it learns to talk every child makes a dreadful nuisance of itself by asking 'Why?'. To stop this nuisance society has invented a marvellous system called education which, for the majority of people, brings to an end their desire to ask that question. The few failures of this system are known as scientists."

2 INPhO-2007

Indian National Physics Olympiad - 2007

INPhO-2007 Jan. 28, 2007

Maximum Marks: **80**

1. The polar coordinates of a particle of mass m moving in a trajectory under the influence of a force $\vec{F}$ are given by: $r = at$ and $\theta = \omega t$, where a and ω are constants. Note acceleration in polar coordinates is

$$a_r = \ddot{r} - r\dot{\theta}^2,\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

   (a) State the momentum vector $\vec{p}$ and the force vector $\vec{F}$.

   (b) Evaluate the work done $\Delta W = \int \vec{F} \cdot d\vec{r}$ explicitly if the initial radial distance of the particle is negligible and the final distance is r.

   (c) Sketch the trajectory. **[6]**

2. A small spherical ball undergoes an elastic collision with a rough horizontal surface. Before the collision, it is moving at an angle θ to the horizontal (see Fig. (8)). You may assume that the frictional force obeys the law $f = \mu N$ during the contact period, where N is the normal reaction on the ball and μ is the coefficient of friction.

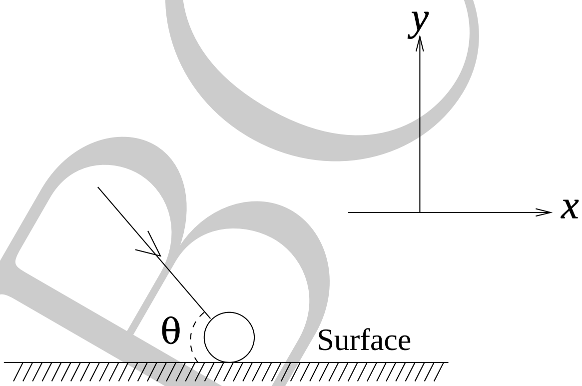


Figure 8: Problem 2

   (a) Obtain $\theta_m(\mu)$ so that the subsequent horizontal range of the ball after leaving the horizontal surface is maximized.

   (b) State the allowed range of θ_m.

[10]

3. A cylindrical block of length 0.4 m and uniform area of cross section 0.04 m^2 is placed in concentric contact with a metal disc of mass 0.4 kg and of the same cross section (see Fig. (9)). The left face (A) of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is $\theta_i = 280.0$ K . If the thermal conductivity of the material of the cylinder is 10 W·m^{-1}·K^{-1} and the specific heat of the material of the disc is

$$C = C_0[1 + \alpha(\theta - \theta_i)]$$

where C_0= 600.0 J·kg^{-1}·K^{-1} and $\alpha = 0.010$ K^{-1}, then:

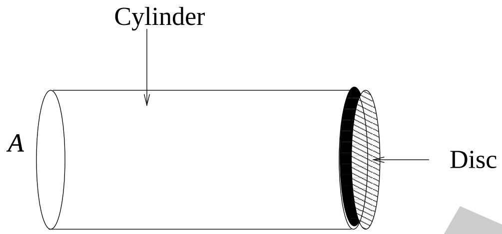


Figure 9: Problem 3

(a) How long will it take for the temperature of the disc to increase to 340 K? Assume that there is no heat loss from the disc.

(b) Repeat the exercise of part (3a) if the specific heat of the disc was $C = C_0$, i.e. temperature independent.

(c) Which process, (3a) or (3b) takes longer time? Why?

Assume that no heat is lost by radiation or convection and that the process of heat transfer is solely conduction. **[10]**

4. A transparent sphere of radius R and refractive index n is at rest on a horizontal surface. A ray of light is incident parallel to the vertical diameter and at a distance d from it.

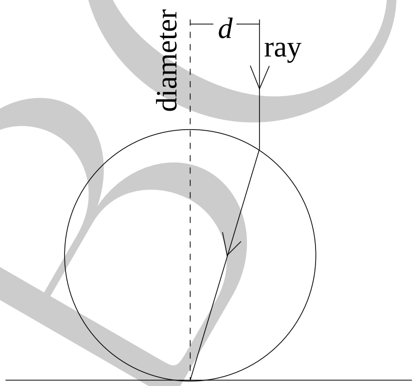


Figure 10: Problem 4

(a) Obtain an expression for d in terms of refractive index n and radius R such that the ray intersects the diameter at the point of emergence (see Fig. (10)).

(b) What is the allowed range of n for the above possibility to occur?

[8]

5. (a) Find the electric field due to an infinite line of charge with linear charge density λ at a distance r from the line.

(b) Using a point at perpendicular distance a from the line charge (i.e. $r = a$) as a reference, find the potential at a distance r from the line.

(c) Now two line charges, with densities λ and $-\lambda$ are kept distance $2d$ apart as shown in Fig. (11). Consider a plane perpendicular to the line charges (e.g. the plane of this paper). Obtain explicit expression for the equipotential lines in this plane.

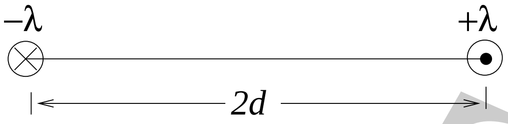


Figure 11: Problem 5

(d) Make a clear plot of these equipotential lines and comment briefly on them. State the shape of the equipotential surfaces.

(e) Now these line charges start moving parallel to each other with speed v. Obtain the speed at which the magnitudes of electric and magnetic forces are equal to each other.

[10]

6. An equilateral triangle of side S carrying a current I_1 is placed with its base at a distance a from an infinite straight wire carrying a current I_2 parallel to the base (see Fig. (12)).

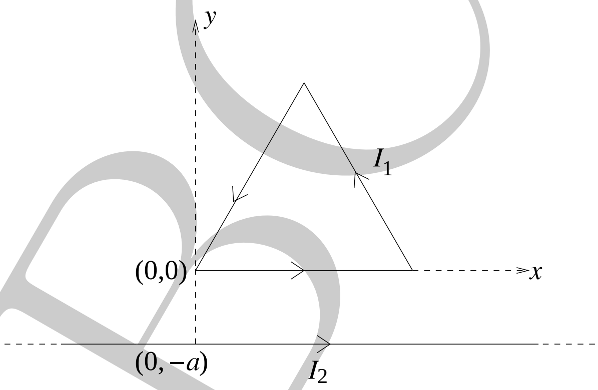


Figure 12: Problem 6

(a) Find the force on the triangle.

(b) Sketch the magnitude of this force as a function of S/a.

[6]

7. A narrow beam of monochromatic light from source S of wave length 6000.0 Å moves along the positive x-axis and is incident on mirror M. The area vector of M is $0.04(-\hat{\imath} + \hat{\jmath})$ m^2. The mirror has reflectivity unity, in other words the mirror is a perfect reflector. An electrically insulated metal surface of total area 0.04 m^2 is placed parallel to x-axis and above the mirror to receive the reflected beam (see Fig. (13)). The work function of the metal is 1.90 eV, its photoelectric efficiency is 10.0% and generated photoelectrons are immediately removed from the neighbourhood. The power of the source is 60.0 W. Assume the metal surface to be large and ignore edge effects. Find out:

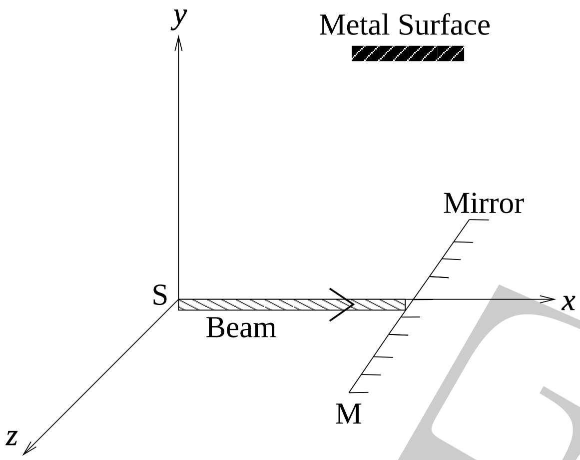


Figure 13: Problem 7

(a) The force exerted by the beam on the mirror.

(b) The surface charge density on the metal surface after 10 seconds.

(c) The energy density due to the electric field after 10 seconds.

(d) The range of kinetic energy of emitted electrons.

[10]

8. Height of the Atmosphere

Consider a simplified model for the height to which the atmosphere extends above the earth's surface. In this model the atmosphere consists of the diatomic gases oxygen and nitrogen in the proportion 21:79 respectively. We assume that the atmosphere is an ideal gas and air processes are adiabatic.

(a) Obtain an expression for the lapse rate Γ (change in temperature T with height z above the earth's surface) in terms of γ, R, g and m_a. Here γ is the ratio of specific heat at constant pressure to specific heat at constant volume; g, the acceleration due to gravity; R, the gas constant; and m_a, the relevant atomic mass.

(b) What is the change in temperature when we ascend a height of one kilometer?

(c) Consider the above model and express the pressure as a function of the height z, the lapse rate Γ and the constants $\{m_a, g, \text{and } R\}$. You may assume that at $z = 0$, the surface temperature $T = T_0$ and pressure $p = p_0$.

(d) According to this model what is the height to which the atmosphere extends? Take $T_0 = 300$ K and $p_0 = 1$ atm.

[10]

9. The Metal Detector

We consider a simple model of the metal detector with a coil (field coil) of radius R_f and concentric and coplanar smaller coil (called the pick-up coil) of radius R_p. The number of turns in the field and pick-up coils are N_f and N_p respectively. A sinusoidal current $I(t)$ is passed through field coil.

(a) State the magnetic field B at the centre of the set-up due to $I(t)$.

(b) We approximate the magnetic field throughout the interior of the smaller coil by the magnetic field calculated in part (9a). Obtain an expression for the induced emf in the pick-up coil.
[Note that this approximation underestimates the flux by about the 10%.]

(c) Given the following values:
$f = 5000$ Hz
maximum current $I_0 = 0.5$ A
maximum induced emf $E_0 = 0.25$ V
$R_p = 0.025$ m
$R_f = 0.05$ m.
Calculate the product $N_p N_f$.

(d) What is the mutual inductance on the field coil due to the pick up coil?

(e) The optimization problem is to use the least amount of wire with the given quantities in part (9c) being kept fixed. Under these constraints determine the allowed ranges of N_p and N_f individually.

(f) Qualitatively describe what happens to the induced emf when you place small disks of the following material at the centre of the pick-up coil:

i. Iron
ii. Wood
iii. Copper

[10]

Metal Detector: The operation of metal detectors is based upon the principle of electromagnetic induction. Metal detectors contain one or more inductor coils that are used to interact with metallic elements which are often hidden or invisible. A pulsing current is applied to the coil, which then induces a magnetic field. When the magnetic field of the coil moves across metal, such as a coin, the field induces electric currents (called eddy currents) in the coin. The eddy currents induce their own magnetic field which generates an opposite current in the coil, which in turn induces a signal indicating the presence of metal.

3 INPhO-2008

Indian National Physics Olympiad - 2008

INPhO-2008 Feb. 03, 2008

Maximum Marks: **80**

1. We define three quantities as follow:

$$A = m_e c^2,\ B = h/m_e c,\ C = e^2/2\epsilon_0 ch$$

where m_e is electron mass and other symbols have their usual meanings. For the hydrogen atom, express the radius of the n^{th} Bohr orbit r_n, the energy level E_n, and the Rydberg constant R in terms of any two of $\{A,\ B,\ C\ \}$.

[5]

2. Consider a ball which is projected horizontally with speed u from the edge of a cliff of height H as shown in the Fig. (14). There is air resistance proportional to the velocity in both x and y direction i.e. the motion in the x (y) direction has air resistance with the deceleration given by the $c\,v_x$ ($c\,v_y$) where c is the proportionality constant and $v_x(v_y)$ is the component of the instantaneous velocity in the x (y) direction. Take the downward direction to be negative. The acceleration due to gravity is g. Take the origin of the system to be at the bottom of the cliff as shown in Fig. (14).

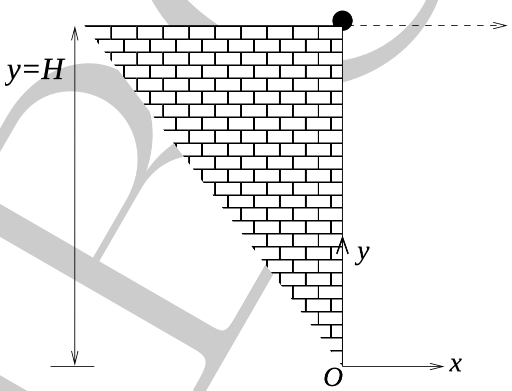


Figure 14: Problem 2

(a) Obtain expressions for $x(t)$ and $y(t)$.

(b) Obtain the expression for the equation of trajectory.

(c) Make a qualitative, comparative sketch of the trajectories with and without air resistance.

(d) Given that height of cliff is $H = 500$ m and $c = 0.05\ \mathrm{s}^{-1}$, obtain the approximate time in which the ball reaches the ground. Take $g = 10\ \mathrm{m{\cdot}s}^{-2}$.

[12]

3. **Free Standing Tower**

Consider a tower of constant density (ρ) and cross sectional area (A) (see Fig.

(15)) at the earth's equator. The tower has a counter weight at one end. It is free standing. In other words its weight is balanced by the outward centrifugal weight so that it exerts no force on the ground beneath it and tension in the tower is zero at both ends. Consider the earth to be an isolated heavenly body and ignore gravitational effects due to the other heavenly bodies such as moon. Further assume that there is no bending of the tower.

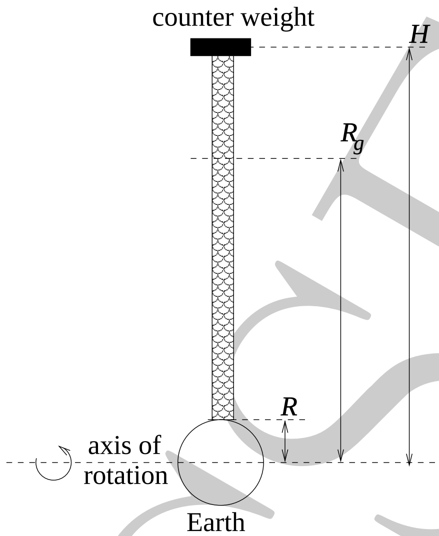


Figure 15: Problem 3

(a) Draw the free body diagram of the small element of this tower at distance r from the centre of the earth.

(b) Let $T(r)$ be the tensile stress (tension per unit area) in the tower. Use Newton's equations to write down the equation for $dT(r)/dr$ in terms of G, ρ, geostationary height R_g from the earth's centre and earth's mass M.

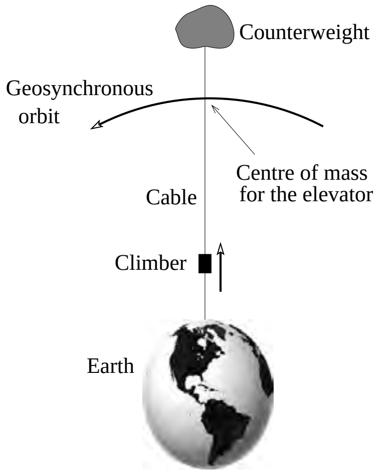
(c) Taking the boundary condition ($T(R) = T(H) = 0$), obtain the height of tower H in terms of R and R_g. Note that R is the radius of earth. Calculate the value of H.

(d) The tensile stress in the tower changes as we move from $r = R$ to $r = H$. Sketch this tensile stress $T(r)$.

(e) Steel has density of $\rho = 7.9 \times 10^3$ kg·m^{-3}. Its breaking tensile strength is 6.37 GPa. Calculate the maximum stress in the tower. State if a tower made of steel would be feasible.

Note: $M = 5.98\times10^{24}$ kg; $R = 6370$ km ; $R_g = 42\ 300$ km

[12]



Space elevator: The space elevator seems like an idea out of a science fiction movie. Put simply its a giant elevator from earth running up to a satellite in space. As crazy as it sounds, a lot of people believe it could work. The technology is based on nanotubes, and they believe that they could create a ribbon cable that could hold a tremendous amount of weight. A runner car will then go up and down on this cable.

4. Two identical walls, each of width w $(= 0.01 \text{ m})$, are separated by a distance d $(= 0.10 \text{ m})$ as shown in Fig. (16). Temperatures of the external face of the walls are fixed (T_1 and T_2, $T_2 > T_1$). Coefficient of thermal conductivity of wall is $k_w = 0.72 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$. We define

$$T_0 = \frac{T_1 + T_2}{2}, \quad \Delta = T_2 - T_1 \quad \text{and} \quad \delta = T'' - T' \tag{1}$$

where T' and T'' are the temperatures of the internal face of the walls 1 and 2 respectively. Then δ will depend on the type of heat transfer process in central region (of width d) between the walls i.e. on the conduction, radiation or convection heat transfer. Assume that the heat transfer is a steady state process.

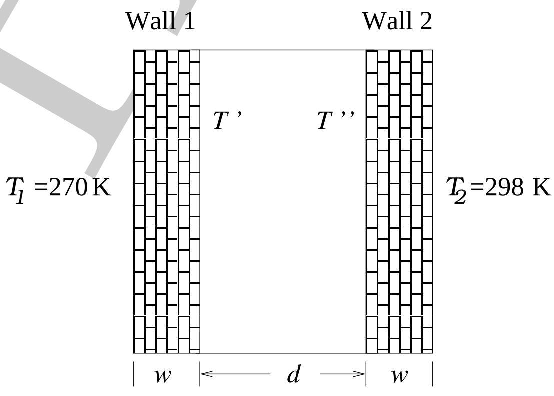


Figure 16: Problem 4

(a) Write down the expression for heat transfer flux q_w (W·m^{-2}) inside the wall 1 in terms of k_w, T_1, T', and w. Similarly also write the expression for wall 2.

(b) Rewrite q_w in terms of $\Delta,\ \delta,\ k_w$, and w.

As mentioned above, in the central region between the walls, heat is transmitted by conduction, convection and radiation. Also due to the steady state process, the corresponding fluxes are equal to q_w. In what follows we will calculate the heat transfer fluxes between the walls due to these three processes each of these processes being considered separately.

Radiation process will take place without the presence of material medium in the central region between the walls. We assume that the central region between the walls is vacuum. Let ϵ be the emissivity of the walls and E_1 and E_2 be the total heat flux due to radiation from wall 1 to 2 and vice versa. Thus $E_1 = \epsilon\sigma T'^4 + (1-\epsilon)E_2$ where σ is the Stefan-Boltzmann constant. Similarly one may write the equation for E_2.

(c) The net heat transfer is $q_r = E_2 - E_1$. Write the expression for q_r in terms of $\epsilon,\ T''$, and T'.

(d) Rewrite q_r in terms of $\{k_w,\ \Delta,\ T_0,\ \sigma,\ \epsilon \text{ and } w\}$.
[Hint: Eliminate δ using $\delta^2 << {T_0}^2$.]

(e) Calculate q_r if $\epsilon = 0.9$.

In the following two parts we are considering only convection between the walls.

(f) Now we assume that central region is filled with air of coefficient of thermal conductivity k_a. In this condition, convected heat transfer between walls will take place. Equation for flux due to this process is given by

$$q_{cv} = \frac{N_u\, k_a}{d}(T'' - T')$$

where N_u is called the Nusselt number and for the given system $N_u = 6.4$. Due to the steady state nature of the process $q_w = q_{cv}$. Express q_{cv} in terms of $\{k_w,\ k_a,\ \Delta,\ w,\ d,\text{ and } N_u\}$.

(g) Calculate the value of q_{cv} if $k_a = 0.026$ W·m^{-1}·K^{-1}.

(h) Instead of air, the central region is now filled with sheathing material having coefficient of thermal conductivity k_s. Hence heat transfer will take place by conduction between walls. Express heat transfer flux q_{cd} in terms of $\{k_s,\ k_w,\ d,\ w,\text{ and } \Delta\}$. We assume that no radiation passes through sheathing material.

(i) Taking $k_s = 0.05$ W·m^{-1}·K^{-1}, calculate the value of q_{cd}.

(j) Considering all possible heat transfer process in the central region between the walls, which insulation (sheathing, air, or vacuum) is the most efficient?

[16]

5. Sunlight falls on the convex surface of the plano - convex lens of aperture 0.080 m. The radius of curvature of the convex surface of the lens is 0.100 m. The refractive indices of the material of the lens for extreme red and violet colours of sunlight are 1.600 and 1.700 respectively. [Given that: Radius of the Sun $= 6.96 \times 10^8$m, Distance between Sun and Earth $= 1.5 \times 10^{11}$m.]

   (a) Calculate the positions of the observed image of the Sun with violet and red centre.

   (b) Calculate the sizes of the observed image of the sun with violet and red centre.

[10]

6. **Determination of The Speed of Light**:
The speed of light maybe determined by an electrical circuit using low frequency ac fields only. Consider the arrangement shown in the Fig. (17). A sinusoidally varying voltage $V_0 \cos(2\pi f t)$ is applied to a parallel plate capacitor C_1 of radius a and separation s and also to the capacitor C_2. The charge flowing into and out of C_2 constitutes the current in the two rings of radii b and separation h. When the voltage is turned off the two sides (the capacitor C_1 on one side and the rings on the other) are exactly balanced. Ignore wire resistance, inductance and gravitational effects.

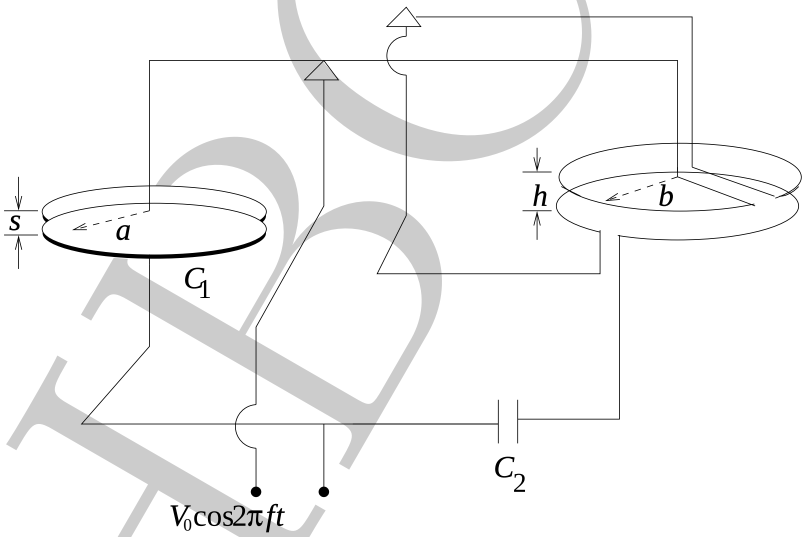


Figure 17: Problem 6

   (a) Obtain an expression for the time-averaged force between the plates of C_1.

   (b) Obtain an expression for the time-averaged force between the rings. The magnetic force between the two rings maybe approximated by those due to long straight wires since $b >> h$.

   (c) Assume that C_2 and the various distances are so adjusted that the time-averaged downward force on the upper plate of C_1 is exactly balanced by the time-averaged downward force on the upper ring. Under these conditions obtain an expression for the speed of light.

(d) Numerically estimate the speed of light given that: $a = 0.10$ m, $s = 0.005$ m, $b = 0.50$ m, $h = 0.02$ m, $f = 60.0$ Hz, $C_1 = 1.00$ nF (nano-farad) and $C_2 = 632$ μF (micro-farad).
[Hint: Not all the given quantities are required to obtain the estimate.]

[12]

7. An N turn metallic ring of radius a, resistance R, and inductance L is held fixed with its axis along a spatially uniform magnetic field $\vec{B}$ whose magnitude is given by $B_0 \sin(\omega t)$.

(a) Set up the emf equation for the current i in the ring.

(b) Assuming that in the steady state i oscillates with the same frequency ω as the magnetic field, obtain the expression for i.

(c) Obtain the force per unit length. Further obtain its oscillatory part and the time-averaged compressional part.

(d) Calculate the time-averaged compressional force per unit length given that $B_0 = 1.00$ tesla, $N = 10$, $a = 10.0$ cm, $\omega = 1000.0$ rad·s^{-1}, $R = 10.0$ Ω, $L = 100.0$ mH.

(e) Answer the following two questions without providing rigorous justification:

i. For $\omega/2\pi = 60$ Hz, the ring emits a humming sound. What is the frequency of this sound?

ii. A capacitor is included in the circuit. How does this affect the force on the ring?

[13]

4 INPhO-2009

Indian National Physics Olympiad - 2009

INPhO-2009 Feb. 01, 2009
Maximum Marks: **70**

Note: Questions 1-38 is a set of multiple choice questions. Only one of the given choices is the best choice. Select this most appropriate choice.

1. A block of weight 200 N is at rest on a rough inclined plane of inclination angle $\theta = 30^0$. The inclined plane is at rest in the earth's inertial frame. Then the magnitude of the force the plane exerts on the block is

   (a) $100\sqrt{3}$ N.
   (b) 100 N
   (c) 200 N
   (d) zero.

2. A spatially uniform magnetic field $\vec{B}$ exists in the circular region S and this field is decreasing in magnitude with time at a constant rate (see Fig. (18)).

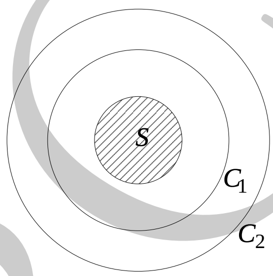


Figure 18: Problem 2

   The wooden ring C_1 and the conducting ring C_2 are concentric with the magnetic field. The magnetic field is perpendicular to the plane of the figure. Then

   (a) there is no induced electric field in C_1.
   (b) there is an induced electric field in C_1 and its magnitude is greater than the magnitude of the induced electric field in C_2.
   (c) there is an induced electric field in C_2 and its magnitude is greater than the induced electric field in C_1.
   (d) there is no induced electric field in C_2.

3. During negative β decay, an anti-neutrino is also emitted along with the ejected electron. Then

   (a) only linear momentum will be conserved.

   (b) total linear momentum and total angular momentum but not total energy will be conserved.

   (c) total liner momentum and total energy but not total angular momentum will be conserved.

(d) total linear momentum, total angular momentum and total energy will be conserved.

4. Five identical balls each of mass m and radius r are strung like beads at random and at rest along a smooth, rigid horizontal thin rod of length L, mounted between immovable supports (see Fig. (19)).

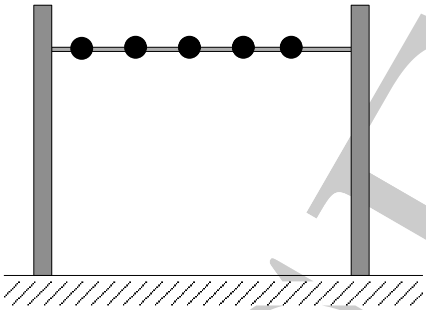


Figure 19: Problem 4

Assume $10r < L$ and that the collision between balls or between balls and supports are elastic. If one ball is struck horizontally so as to acquire a speed v, the magnitude of the average force felt by the support is

(a) $\dfrac{5mv^2}{L-5r}$

(b) $\dfrac{mv^2}{L-10r}$

(c) $\dfrac{5mv^2}{L-10r}$

(d) $\dfrac{mv^2}{L-5r}$

5. In Young's double slit experiment, one of the slits is wider than the other, so that the amplitude of the light from one slit is double that from the other slit. If I_m be the maximum intensity, the resultant intensity when they interfere at phase difference ϕ is given by

(a) $\dfrac{I_m}{3}\left(1+2\cos^2\dfrac{\phi}{2}\right)$

(b) $\dfrac{I_m}{5}\left(1+4\cos^2\dfrac{\phi}{2}\right)$

(c) $\dfrac{I_m}{9}\left(1+8\cos^2\dfrac{\phi}{2}\right)$

(d) $\dfrac{I_m}{9}\left(8+\cos^2\dfrac{\phi}{2}\right)$

6. A point luminous object (O) is at a distance h from front face of a glass slab of width d and of refractive index n. On the back face of slab is a reflecting plane mirror. An observer sees the image of object in mirror (see Fig. (20)).

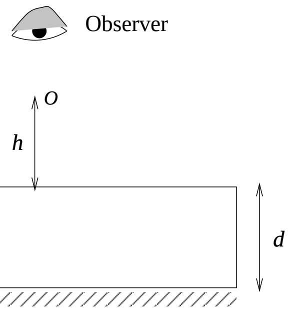


Figure 20: Problem 6

Distance of image from front face as seen by observer will be

(a) $h + \dfrac{2d}{n}$

(b) $2h + 2d$

(c) $h + d$

(d) $h + \dfrac{d}{n}$

7. A uniform wire of diameter 0.04 cm and length 60 cm made of steel (density 8000 kg·m^{-3}) is tied at both ends under a tension of 80 N. Transverse vibrations of frequency about 700 Hz will be predominant if the wire is plucked at

(a) 15 cm and held at 30 cm.

(b) 10 cm and held at 20 cm.

(c) 30 cm.

(d) 20 cm and held at 40 cm.

8. Consider a circle of radius R. A point charge lies at a distance a from its centre and on its axis such that $R = a\sqrt{3}$. If electric flux passing through the circle is ϕ then the magnitude of the point charge is

(a) $\sqrt{3}\epsilon_0\phi$

(b) $2\epsilon_0\phi$

(c) $4\epsilon_0\phi/\sqrt{3}$

(d) $4\epsilon_0\phi$

9. A uniform tube 60 cm long, stands vertically with lower end dipping into water. When its length above water is 14.8 cm and successively again when it is 48 cm, the tube resonates to a vibrating tuning fork of frequency 512 Hz. The lowest frequency to which this tube can resonate when it is taken out of water is nearly

(a) 275 Hz

(b) 267 Hz

(c) 283 Hz

(d) 256 Hz

10. A binary star has a period (T) of 2 earth years while distance L between its components having masses M_1 and M_2 is four astronomical units. If $M_1 = M_S$ where M_S is the mass of Sun, the mass of other component M_2 is

    (a) $3M_S$
    (b) 7 M_S
    (c) 15 M_S
    (d) M_S

    Note: The earth - sun distance is one astronomical unit.

11. A uniform rod of mass $2M$ is bent into four adjacent semicircles each of radius r all lying in the same plane (see Fig. (21)). The moment of inertia of the bent rod about an axis through one end A and perpendicular to plane of rod is

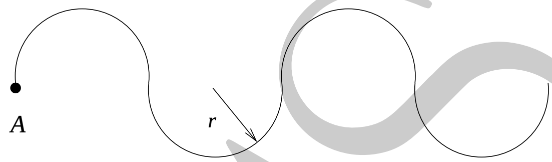


Figure 21: Problem 11

    (a) $22Mr^2$
    (b) $88Mr^2$
    (c) $44Mr^2$
    (d) $66Mr^2$

12. Two pulses on the same string are described by the following wave equations:

$$y_1 = \frac{5}{(3x-4t)^2+2} \quad \text{and} \quad y_2 = \frac{-5}{(3x+4t-6)^2+2}.$$

    Choose the INCORRECT statement.

    (a) Pulse y_1 and pulse y_2 travel along +ve and -ve x axis respectively.
    (b) At $t = 0.75$ s, displacement at all points on the string is zero.
    (c) At $x = 1$ m displacement is zero for all times.
    (d) Energy of string is zero at $t = 0.75$ s.

13. A ray of light enters at grazing angle of incidence into an assembly of five isosceles right-angled prisms having refractive indices $\mu_1,\ \mu_2,\ \mu_3,\ \mu_4$ and μ_5 respectively (see Fig. (22)).

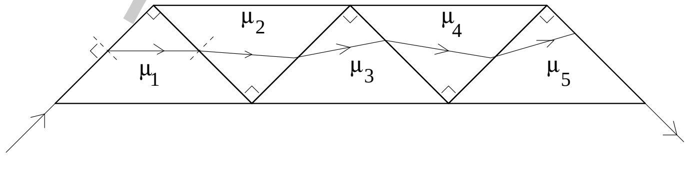


Figure 22: Problem 13

The ray also emerges out at grazing angle. Then

(a) $\mu_1^2 + \mu_3^2 + \mu_5^2 = 1 + \mu_2^2 + \mu_4^2$

(b) $\mu_1^2 + \mu_3^2 + \mu_5^2 = 2 + \mu_2^2 + \mu_4^2$

(c) $\mu_1^2 + \mu_3^2 + \mu_5^2 = \mu_2^2 + \mu_4^2$

(d) none of the above

14. The circuit shown in Fig. (23)) is allowed to reach steady state and then a soft iron core is quickly inserted in the coil such that the coefficient of self inductance changes from L to nL.

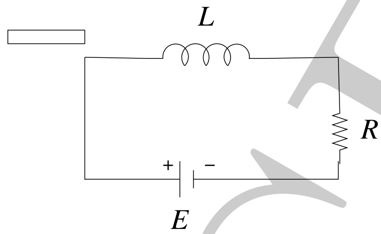


Figure 23: Problem 14

The current in the circuit at the time of complete insertion is

(a) E/R

(b) nE/R

(c) E/nR

(d) zero

15. Consider an infinitely extending gas cloud in space with two "rigid" spherical vacuum cavities (see Fig. (24)).

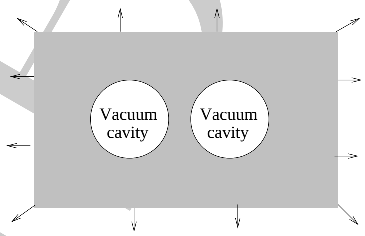


Figure 24: Problem 15

Consider only gravitational forces between gas molecules. Then

(a) the cavities would come closer to each other.

(b) the cavities would move away from each other.

(c) the cavities would be static.

(d) the motion of cavities would depend on the size of cavities.

Questions (16) and (17) are based on Fig. (25) and following information.

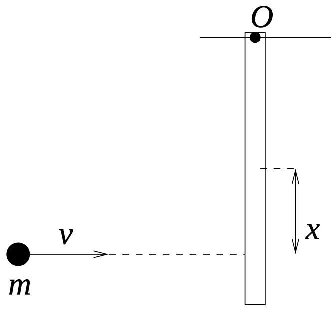


Figure 25: Problems 16 and 17

A rod of mass m and length l is hinged at one end O. A particle of mass m travelling with speed v collides with the rod at a distance x from the centre of mass of the rod such that the reaction force at the hinge is zero.

16. Then for the system

    (a) linear momentum is conserved.

    (b) angular momentum is not conserved about point O.

    (c) Linear momentum is not conserved and angular momentum about point O is conserved.

    (d) the mechanical energy is conserved.

17. Then

    (a) $x = l/6$.

    (b) $x = l/2$.

    (c) $x = l/3$.

    (d) $x = l/4$.

18. Consider a huge charge reservoir at potential V. A spherical capacitor C_1 is brought in contact with the charge reservoir and then removed. Next another spherical capacitor C_2 is brought in contact with C_1 and removed. We repeat this process a large number of times. Assume that potential of reservoir does not change during this exercise. Then the charge on C_2 after a very long time is

    (a) C_2V

    (b) C_1V

    (c) $C_2C_1V/(C_1 + C_2)$

    (d) $(C_1 + C_2)V$

19. A particle of mass 1 kg is taken along the path $ABCDE$ from A to E (see Fig. (26)). The two "hills" are of heights 50 m and 100 m and the horizontal distance AE is 20 m while the path length is 400 m. The coefficient of friction of the surface is 0.1. Take $g = 10$ m·s^{-2} and $\sqrt{3} = 1.73$. The minimum work on the mass required to accomplish this is

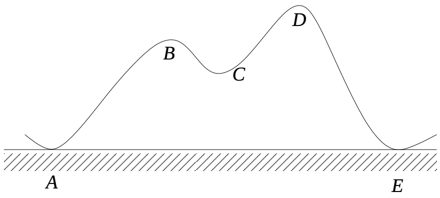


Figure 26: Problem 19

(a) 20 J

(b) 173 J

(c) 400 J

(d) 0 J

20. Two positrons (e^+) and two protons (p) are kept on four corners of a square of side a as shown in Fig. (27). The mass of proton is much larger than the mass of positron. Let q denote the charge on the proton as well as the positron. Then the kinetic energies of one of the positrons and one of the protons respectively after a very long time will be

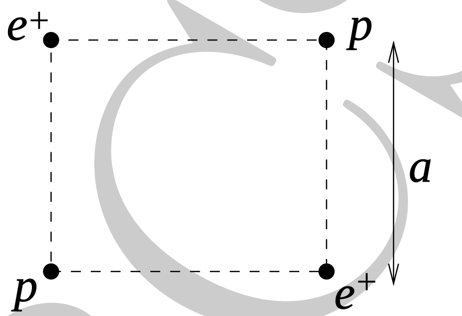


Figure 27: Problem 20

(a) $\dfrac{q^2}{4\pi\epsilon_0 a}\left(1+\dfrac{1}{2\sqrt{2}}\right)$, $\quad \dfrac{q^2}{4\pi\epsilon_0 a}\left(1+\dfrac{1}{2\sqrt{2}}\right)$

(b) $\dfrac{q^2}{2\pi\epsilon_0 a}$, $\quad \dfrac{q^2}{4\sqrt{2}\pi\epsilon_0 a}$

(c) $\dfrac{q^2}{4\pi\epsilon_0 a}$, $\quad \dfrac{q^2}{4\pi\epsilon_0 a}$

(d) $\dfrac{q^2}{2\pi\epsilon_0 a}\left(1+\dfrac{1}{4\sqrt{2}}\right)$, $\quad \dfrac{q^2}{8\sqrt{2}\pi\epsilon_0 a}$

21. An electrostatic field line leaves at angle α from point charge q_1, and connects with point charge $-q_2$ at angle β (see Fig. (28)).

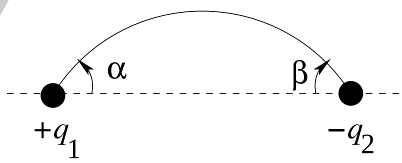


Figure 28: Problem 21

Then the relationship between α and β is

(a) $q_1 \sin^2 \alpha = q_2 \sin^2 \beta$.

(b) $q_1 \tan \alpha = q_2 \tan \beta$.

(c) $q_1 \sin^2 \dfrac{\alpha}{2} = q_2 \sin^2 \dfrac{\beta}{2}$.

(d) $q_1 \cos \alpha = q_2 \cos \beta$.

22. A square metal frame in the vertical plane is hinged at O at its centre (see Fig. (29)).

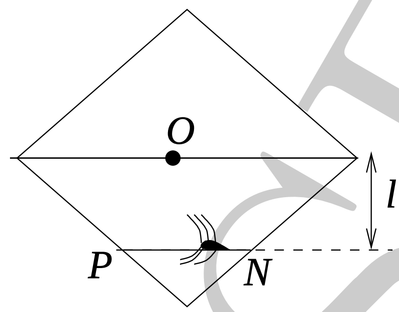


Figure 29: Problem 22

A bug moves along the rod PN which is at a distance l from the hinge, such that the whole frame is always stationary, even though the frame is free to rotate in the vertical plane about the hinge. Then the motion of the bug will be simple harmonic, with time period,

(a) $2\pi\sqrt{l/g}$

(b) $2\pi\sqrt{2l/g}$

(c) $2\pi\sqrt{4l/g}$

(d) $2\pi\sqrt{l/2g}$

[**Hint:** There is a frictional force between the rod and the bug.]

23. A long flexible inextensible rope of uniform linear mass density λ is being pulled on a rough floor with horizontal force $\vec{F}$ in such a way that its lower part is at rest and upper part moves with constant speed v (see Fig. (30)). The magnitude of $\vec{F}$ will be

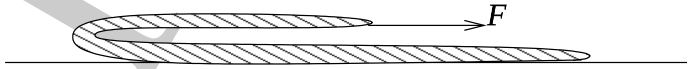


Figure 30: Problem 23

(a) $2\lambda v^2$

(b) λv^2

(c) $\lambda v^2/2$

(d) some function of time and not constant.

24. A particle moving with initial velocity $\vec{v}_i = (3\hat{\imath} + 5\hat{\jmath})$ m·s^{-1} collides with a smooth plane wall placed at some orientation to the particle's trajectory. The resulting velocity of the particle is $\vec{v}_f = (-2\hat{\imath} - \hat{\jmath})$ m·s^{-1}. The coefficient of restitution for this collision is

   (a) 16/33
   (b) 5/34
   (c) 16/45
   (d) 8/45

25. A long straight wire is carrying current I_1 in $+z$ direction. The x-y plane contains a closed circular loop carrying current I_2 and not encircling the straight wire. The force on the loop will be

   (a) $\mu_0 I_1 I_2/2\pi$.
   (b) $\mu_0 I_1 I_2/4\pi$.
   (c) zero.
   (d) depends on the distance of the centre of the loop from the wire.

26. A uniform electric field $\vec{E}$ in the y-direction and uniform magnetic field $\vec{B}$ in the x-direction exists in free space. A particle of mass m and carrying charge q is projected from the origin with speed v_0 along the y-axis. The speed of particle as a function of its y coordinate will be

   (a) $\sqrt{v_0^2 + \dfrac{2qEy}{m}}$
   (b) $\sqrt{v_0^2 - \dfrac{4qEy}{m}}$
   (c) $\sqrt{v_0^2 + \dfrac{qEy}{m}}$
   (d) v_0.

27. The atmospheric pressure on the earth's surface is P in MKS units. A table of area 2 m^2 is tilted at 45^0 to the horizontal. The force on the table due to the atmosphere is (in newtons)

   (a) $2P$
   (b) $\sqrt{2}P$
   (c) $2\sqrt{2}P$
   (d) $P/\sqrt{2}$

28. The shear modulus of lead is 2×10^9 Pa. A cubic lead slab of side 50 cm is subjected to a shearing force of magnitude 9.0×10^4 N on its narrow face (see Fig. (31)).

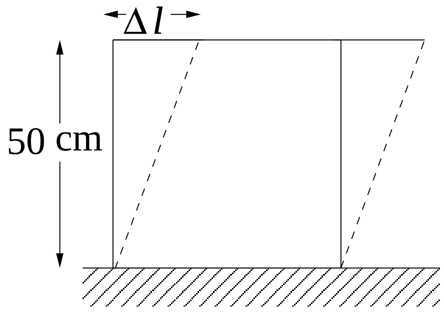


Figure 31: Problem 28

The displacement of the upper edge is δl, where δl is

(a) 2×10^{-3} m
(b) 5×10^{-4} m
(c) 4×10^{-4} m
(d) 9×10^{-5} m

29. In a moving coil galvanometer the number of turns $N = 24$, area of the coil $A = 2 \times 10^{-3}$ m^2, and the magnetic field strength $B = 0.2$ T. To increase its current sensitivity by 25% we

(a) Increase B to 0.30 T
(b) Decrease A to 1.5×10^{-3} m^2
(c) Increase N to 30
(d) None of the above.

30. Which of the following statement is TRUE ?

(a) Sound waves cannot interfere.
(b) Only light waves may interfere.
(c) The de Broglie waves associated with moving particles can interfere.
(d) The Bragg formula for crystal structure is an example of the corpuscular nature of electromagnetic radiation.

31. Two metallic rods AB and BC of different materials are joined together at the junction B (see Fig. (32)). It is observed that if the ends A and C are kept at 100^0 C and 0^0 C respectively, the temperature of the junction B is 60^0 C. There is no loss of heat to the surroundings. The rod BC is replaced by another rod BC' of the same material and length ($BC = BC'$). If the area of cross-section of BC' is twice that of BC and the ends A and C' are maintained at 100^0 C and 0^0 C respectively, the temperature of the junction B will be nearly

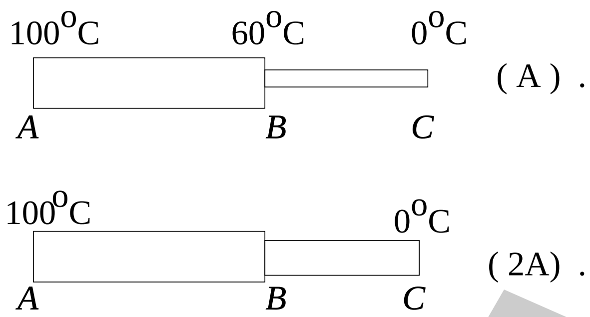


Figure 32: Problem 31

(a) 29^0 C
(b) 33^0 C
(c) 60^0 C
(d) 43^0 C

32. Three closed vessels A, B and C are at the same temperature (T) and contain gases which obey the Maxwellian distribution of velocities. The vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the N_2 molecules in vessel B is V_2 and that of oxygen molecules in A is V_1, the average speed of N_2 molecules in C is

(a) $(V_1 + V_2)/2$
(b) $(V_1 - V_2)/2$
(c) V_2
(d) $\sqrt{(V_1 V_2)}$

33. When a system is taken from state a to state b along the path $a - c - b$ (see Fig. (33)), 60 J of heat flows into the system and 30 J of work are done by the system. Along the path $a - d - b$, if the work done by the system is 10 J, heat flow into the system is

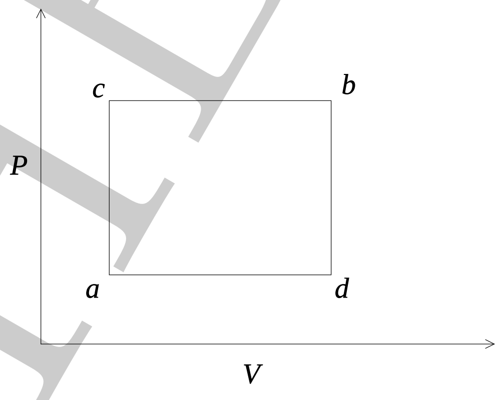


Figure 33: Problem 33

(a) 100 J
(b) 20 J
(c) 80 J
(d) 40 J

34. Two identical piano strings, when stretched with the same tension T_0, have a fundamental frequency of 300 Hz. The tension in one of the strings is increased to $(T_0 + \Delta T)$ and 3 beats per second occur when both strings vibrate simultaneously. $(\Delta T/T_0) \times 100$ is

(a) 2
(b) 3
(c) 1
(d) 4

35. The half life of a certain radioactive material $({}_zX^{100})$ is 6.93×10^6 s. In order to have an activity of 6.0×10^8 disintegrations per second, the amount of material needed is nearly

(a) 10^{-9} kg
(b) 10^{-16} kg
(c) 10^{-6} kg
(d) 10^{-4} kg

36. Sound of frequency 1000 Hz from a stationary source is reflected from an object approaching the source at 30 m·s^{-1}, back to a stationary observer located at the source. The speed of sound in air is 330 m·s^{-1}. The frequency of the sound heard by the observer is

(a) 1200 Hz
(b) 1000 Hz
(c) 1090 Hz
(d) 1100 Hz

37. Current (I) - applied voltage (V) characteristics are shown in Fig. (34). Possible observed plot(s) for a photoelectric setup is (are):

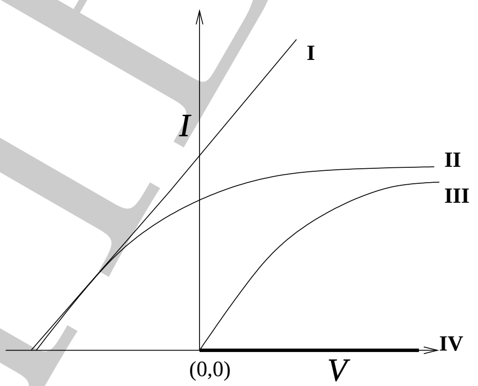


Figure 34: Problem 37

(a) only II.
(b) I and II.
(c) II and III.
(d) II and IV.

38. A triply ionized beryllium (Be^{+++}) has the some orbital radius as the ground state of hydrogen. Then the quantum state n of Be^{+++} is

(a) $n = 1$
(b) $n = 2$
(c) $n = 3$
(d) $n = 4$

39. One mole of gas undergoes a linear process as shown in Fig. (35).

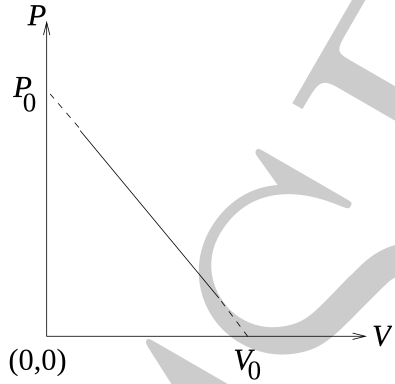


Figure 35: Problem 39

(a) Express P in terms of $\{V,\ V_0,\ P_0\}$.

(b) Assuming that the gas is ideal, obtain the expression for T in terms of gas constant R and $\{V,\ V_0,\ P_0\}$.

(c) Obtain the expression for volume change with temperature (dV/dT) in terms of $\{R,\ V,\ V_0, P_0\}$.

(d) Let T_{max} be the maximum temperature in the process. Express T_{max} in terms of $\{V_0,\ P_0,\ \mathrm{R}\}$.

(e) Sketch the $T - V$ diagram. (T on y-axis and V on x-axis.)

(f) Let $C_p/C_v = \gamma$, where C_p (C_v) is specific heat at constant pressure (volume). Express heat capacity C_v in terms of R and γ.

(g) Using the first law of thermodynamics, obtain the expression for specific heat C for the above linear process in terms of $\{R,\ \gamma,\ V_0,\ V\}$.

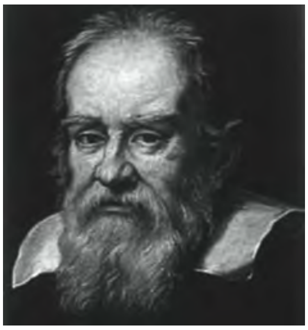
(h) Suppose mixture consists of half mole of mono atomic and half mole of diatomic gas. Obtain γ for this mixture.

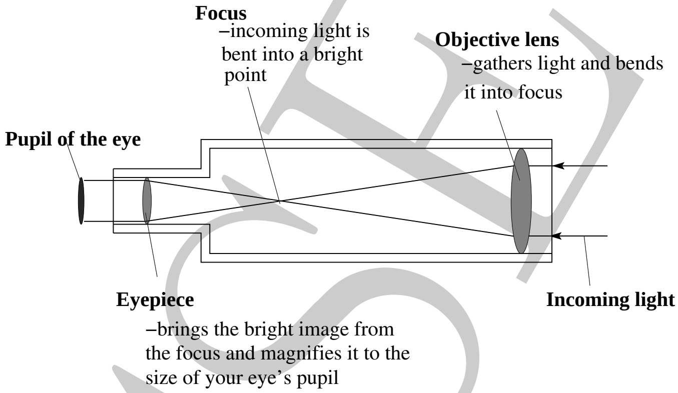
(i) For the mixture described in Part (39h), obtain C in terms of $\{R,\ V,\ V_0\}$.

(j) Plot C/R (on y-axis) vs V/V_0 (x-axis).

[13]

400 years of telescope: Dutch eyeglass-maker Hans Lippershey first tried to patent the telescope in October 1608, and his invention was soon a big hit in Europe as a tool for insider trading. Futures contracts were in vogue, and spying a cargo ship first had financial benefits. The telescope also redefined our universe: In 1608, Earth was the centre of God's perfect Creation. By 1610, Galileo showed that Jupiter had moons, Earth's moon had mountains, and the Catholic church was fallible. Four centuries on, we know we are a mere speck in a universe of wonders.





Galileo Galilei (15 Feb.1564-8 Jan.1642) : Italian natural philosopher, astronomer, and mathematician who applied the new techniques of the scientific method to make significant discoveries in physics and astronomy. His great accomplishments include perfecting (though not inventing) the telescope and consequent contributions to astronomy. He studied the science of motion, inertia, the law of falling bodies, and parabolic trajectories. His formulation of the scientific method parallels the writings of Francis Bacon. His progress came at a price, since his ideas were in conflict with religious dogma. He believed the Earth revolved around the Sun. For this, he was interrogated by the Inquisition, was put on trial, found guilty and sentenced to indefinite imprisonment. For renouncing his former beliefs before the Cardinals that judged him, he was allowed to serve this time instead under house-arrest.
"In questions of science the authority of a thousand is not worth the humble reasoning of a single individual."

Chapter II

Brief Solutions

1 INPhO-2006

1. (a) a=0

   (b) $f_s = 5$ N. and $T \cong 10$ N.

2. (a) $a = \dfrac{\mu g}{3}$

   (b) $t^* = \dfrac{3}{7} \cdot \dfrac{R\,\omega_0}{\mu\,g}$

   (c) $t_{tot} = \dfrac{R\,\omega_0}{\mu\,g}$

3. (a) $\eta = 1 - \dfrac{T_c - T_d}{T_b - T_a}$

   (b) $\eta = 1 - \left(\dfrac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

   (c)

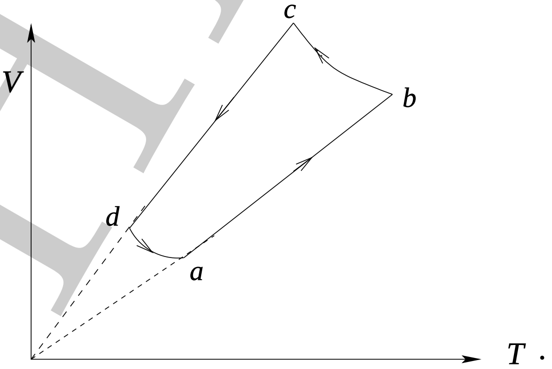


Figure 1: V-T diagram

   (d) $\eta_c = 1 - \dfrac{T_d}{T_b}$,
   Carnot cycle has higher efficiency than the given cycle.

4. Amplitude of the image= 2.78 cm
   The phase difference = 0
   Angle made by the virtual image with the principle axis = 30.26^0

5. (a) $\mu' = \dfrac{\pi\omega\sigma R^4}{4}$ and along the positive x-direction.
   (b) $B = \dfrac{\mu_0\,\omega\,\sigma}{2}\left[\dfrac{R^2+2\,a^2}{\sqrt{R^2+a^2}} - 2\,a\right]$ and along the positive x-direction.
   (c) $B = \dfrac{\mu_0\,\omega\,\sigma R^4}{8a^3}$ and along the positive x-direction.
   (d) $F = -\dfrac{3\,\mu\,\mu_0\,\omega\,\sigma\,R^4}{8\,a^4}$ and direction dictated by the direction of $\vec{\mu}$.

6. (a) $F = 4\pi np\left[\dfrac{\delta^2}{2} - \dfrac{\delta^4}{4\,R^2}\right]$ and direction upwards.
   (b) $m = 6.80 \times 10^{-7}$ kg.

7. (a) $\lambda = 2080$ Å. This wavelength belongs to ultraviolet region of the electromagnetic spectrum.
   (b) Maximum Kinetic Energy $K_{max} = 2.75$ eV
   De Broglie wavelength $\lambda_{dB} = 7.4$ Å

8. (a) $y \geqslant 10^{-18}$
   (b) $F = \dfrac{\eta e^2 y^2 r}{3\epsilon_0 m_p}$
   (c) $H = 1.8 \times 10^{-17}\ \mathrm{s}^{-1}$
   (d) $V = V_0\, e^{3Ht}$
   (e) The numerical value of H does not agree with the known value of Hubble's constant. This is one of the reasons we may discard the Lyttleton Bondi model.

2 INPhO-2007

1. (a) $\vec{p} = m(a\hat{r} + r\omega\hat{\theta})$
   $\vec{F} = m(-r\omega^2\hat{r} + 2a\omega\hat{\theta})$

   (b) $\Delta W = \dfrac{m\omega^2 r^2}{2}$

   (c) Trajectory will be a spiral.

2. (a) $\theta_m = \dfrac{\cot^{-1}(2\mu)}{2}$

   (b) Possible range of θ_m : $\theta_m \in]0, \pi/4[$

3. (a) $t \approx 222$ s

   (b) $t = 166$ s

   (c) Process (3a) takes more time since as the disk heats up, its specific heat also increases and more heat is required to effect a further rise in temperature.

4. (a) $d^2 = R^2 \left(1 - \dfrac{(n^2-2)^2}{4}\right)$

   (b) The allowed range of n is $n \in]\sqrt{2}, 2[$

5. (a) $E(r) = \dfrac{\lambda}{2\pi\epsilon_0 r}$

   (b) $V(r) = -\dfrac{\lambda}{2\pi\epsilon_0} \ln\left(\dfrac{r}{a}\right)$

   (c) $\left(x - d\dfrac{k^2+1}{k^2-1}\right)^2 + y^2 = \dfrac{4d^2k^2}{(k^2-1)^2}$ This is an equation of a circle with centre at $\left(d\dfrac{k^2+1}{k^2-1}, 0\right)$ and radius given by $\dfrac{2dk}{|k^2-1|}$. The constant k is related to the "equipotential" V_0.

   (d) See Fig. (2) and part (c)

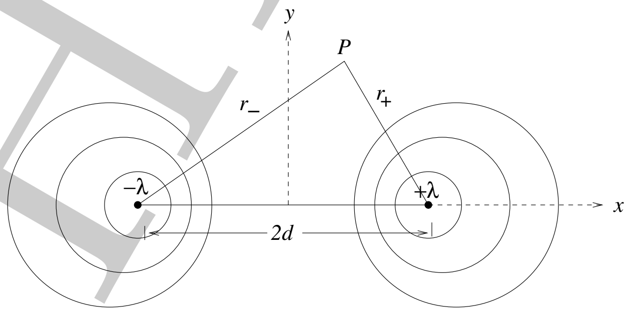


Figure 2: Problem 5 (d)

   (e) $v = 1/\sqrt{\epsilon_0\mu_0} = c$

6. (a) $F = \dfrac{\mu_0 I_1 I_2}{\sqrt{3}\pi}\left[\dfrac{\sqrt{3}S}{2a} - \ln\left(1 + \dfrac{\sqrt{3}S}{2a}\right)\right]$ and direction downwards.

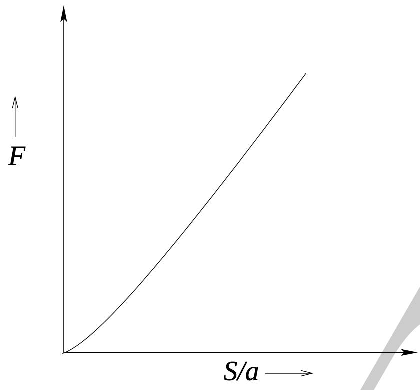


Figure 3: Problem 6 (b)

(b) See Fig. (3).

7. (a) $|\vec{F}| = 2.82 \times 10^{-7}$ N
   (b) $\sigma = 7.27 \times 10^{2}\,\mathrm{C{\cdot}m^{-2}}$
   (c) Energy density after 10 s$= 2.93 \times 10^{16}\,\mathrm{J{\cdot}m^{-3}}$
   (d) The range of kinetic energy of electrons is from 0 to 0.16 eV.

8. (a) $\Gamma = -\dfrac{(\gamma-1)}{\gamma}\dfrac{m_a g}{R}$
   (b) $\Gamma = -0.01\ \mathrm{K{\cdot}m^{-1}}$ (i.e. 1^0 C decrease for every 100 m)
   (c) $p = p_0\left(\dfrac{T_0 - \Gamma\, z}{T_0}\right)^{m_a g/R\Gamma}$
   (d) Substituting the given values in the above equation, the height of the atmosphere is approximately 30 km.

9. (a) $B = \dfrac{\mu_0}{2}\dfrac{N_f}{R_f}I(t)$ and direction given by right hand thumb rule.
   (b) $|\varepsilon| = \dfrac{\mu_0}{2}\dfrac{N_pN_f}{R_f}\pi {R_p}^2\dfrac{dI(t)}{dt}$
   (c) $N_pN_f = 645$
   (d) $M = \dfrac{\mu_0}{2}\dfrac{N_pN_f}{R_f}\pi {R_p}^2 = 1.59 \times 10^{-5}$ H
   (e) $N_f = 18$ turns, $N_p = 36$ turns
   (f) Induced emf in case of
      i. Iron : will increase.
      ii. Wood : no appreciable change.
      iii. Copper: decrease.

3 INPhO-2008

1. $r_n = \dfrac{n^2}{2\pi}\dfrac{B}{C}$; $E_n = -\dfrac{1}{2n^2}\,AC^2$; $R = \dfrac{C^2}{2B}$

2. (a) $x = \dfrac{u}{c}\,(1 - e^{-ct})$; $y = H + \dfrac{g}{c^2} - \dfrac{g}{c}\left[\dfrac{1}{c}\,e^{-ct} + t\right]$

   (b) $y = H - \dfrac{g\,x^2}{2\,u^2} - \dfrac{g\,x^3\,c}{3\,u^3}$

   (c) See Fig. (4)

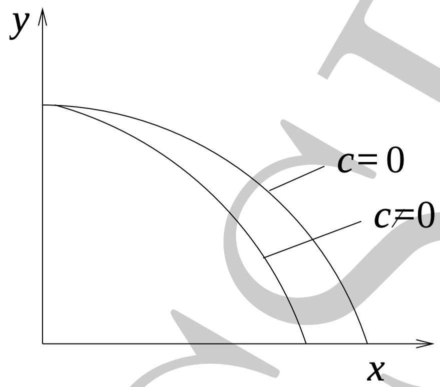


Figure 4: Problem 2 (c)

   (d) $t = 11.1$ s

3. (a) Free Body Diagram, (Fig. (5))

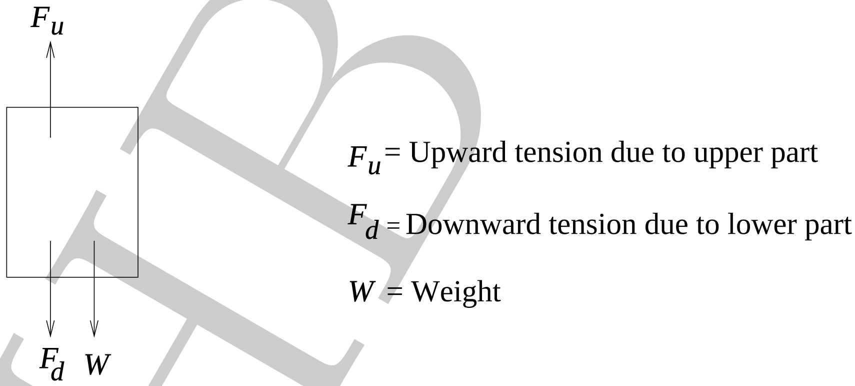


Figure 5: Problem 3 (a)

   (b) $\dfrac{dT}{dr} = GM\rho\left[\dfrac{1}{r^2} - \dfrac{r}{R_g^3}\right]$

   (c) $H = \dfrac{R}{2}\left[\sqrt{1 + \dfrac{8\,R_g^3}{R^3}} - 1\right] = 1.51 \times 10^5$ km

(d) See Fig. (6)

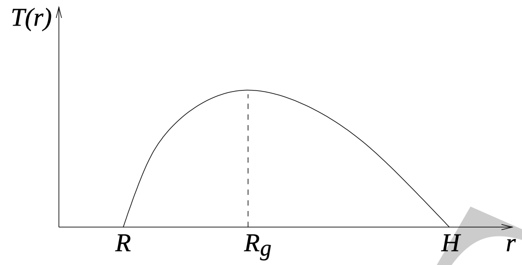


Figure 6: Problem 3 (d)

(e) Maximum stress will be at $r = R_g$

$$T(R_g) = 379 \;\; \text{GPa}$$

Steel has tensile strength 6.37 GPa which is less than 379 GPa. Hence it will not be feasible.

4. (a) For wall 1, $q_w = \dfrac{k_w}{w}\,(T_2 - T'')$

For wall 2, $q_w = \dfrac{k_w}{w}\,(T' - T_1)$

(b) $q_w = \dfrac{k_w}{w}\,\dfrac{(\Delta - \delta)}{2}$

(c) $q_r = \dfrac{\epsilon\,\sigma}{(2-\epsilon)}\,(T''^4 - T'^4)$

(d) $q_r = \dfrac{k_w}{w}\left(\dfrac{\Delta\,4c\,T_0^3}{1 + 8c\,T_0^3}\right)$

(e) $q_r = 107.22$ W·m^{-2}

(f) $q_{cv} = \dfrac{N_u\,k_a\,\Delta\,k_w}{k_w\,d + 2\,w\,k_a\,N_u}$

(g) $q_{cv} \simeq 46.5$ W·m^{-2}

(h) $q_{cd} = \dfrac{k_s\,k_w\,\Delta}{2\,wk_s + k_w\,d}$

(i) $q_{cd} = 13.8$ W·m^{-2}

(j) Sheathing

5. (a) Position of the image with violet centre $\mathrm{f}_V = 14.3$ cm ;
Position of the image with red centre $\mathrm{f}_R = 16.7$ cm ;
Size of the image with violet centre $\mathrm{I}_V \simeq 0.64$ mm ;
Size of the image with red centre $\mathrm{I}_R \simeq 0.74$ mm

6. (a) $\langle F_e\rangle = \dfrac{C_1^2\,V_0^2}{4\,\pi\epsilon_0\,a^2}$

(b) $\langle F_m\rangle = \dfrac{\mu_0\,b}{2\,h}C_2^2V_0^2(2\pi f)^2$

(c) $c = (2\pi)^{3/2}\, a \left(\dfrac{b}{h}\right)^{1/2} \dfrac{C_2}{C_1}\, f$

(d) $c = 2.99 \times 10^{8}\ \mathrm{m \cdot s^{-1}}$

7. (a) $i\,R + L\,\dfrac{di}{dt} = -N\pi a^2 B_0\,\omega \cos\omega t$

(b) $i = \dfrac{N\,\pi\,a^2 B_0\,\omega(R\cos\omega t + \omega L\sin\omega t)}{R^2 + \omega^2\,L^2}$

(c) $\dfrac{dF}{dl} = -\dfrac{NB_0^2\,\pi a^2\omega}{R^2+\omega^2L^2}(R\,\sin\omega t\,\cos\omega t + \omega\,L\sin^2\omega t)$

$\left.\dfrac{dF}{dl}\right|_{av} = -\dfrac{NB_0^2\,\pi a^2\omega^2 L}{2(R^2+\omega^2L^2)}$

$\left.\dfrac{dF}{dl}\right|_{osc} = -\dfrac{NB_0^2\,\pi a^2\omega}{2(R^2+\omega^2L^2)}(R\sin 2\omega t - \omega L\cos 2\omega t)$

(d) $\left.\dfrac{dF}{dl}\right|_{av} = 1.55\ \mathrm{N{\cdot}m^{-1}}$

(e) i. The frequency of the sound is 120 Hz.

ii. The compressional force is lessened and may even become tensile.

4 INPhO-2009

(1) c	(6) a	(11) c	(16) a	(21) c	(26) a	(31) d	(36) a
(2) b	(7) b	(12) d	(17) a	(22) a	(27) a	(32) c	(37) d
(3) d	(8) d	(13) b	(18) a	(23) c	(28) d	(33) d	(38) b
(4) b	(9) b	(14) c	(19) d	(24) c	(29) c	(34) a	
(5) c	(10) c	(15) b	(20) d	(25) c	(30) c	(35) a	

(39) (a) $\dfrac{P}{P_0} + \dfrac{V}{V_0} = 1$ (valid for $V < V_0$, $P < P_0$)

(b) $T = \dfrac{P_0 V}{R}\left(1 - \dfrac{V}{V_0}\right)$

(c) $\dfrac{dV}{dT} = \dfrac{RV_0}{P_0(V_0 - 2V)}$

(d) $T_{max} = \dfrac{P_0 V_0}{4R}$

(e)

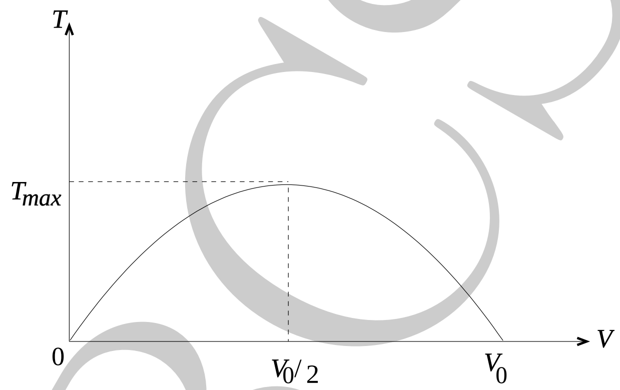


Figure 7: Problem 39 (e)

(f) $C_v = \dfrac{R}{\gamma - 1}$

(g) $C = \dfrac{R}{\gamma - 1} + \dfrac{(V_0 - V)R}{(V_0 - 2V)}$

(h) $\gamma = \dfrac{3}{2}$

(i) $C = R\dfrac{\left(3 - \dfrac{5V}{V_0}\right)}{\left(1 - \dfrac{2V}{V_0}\right)}$

(j)

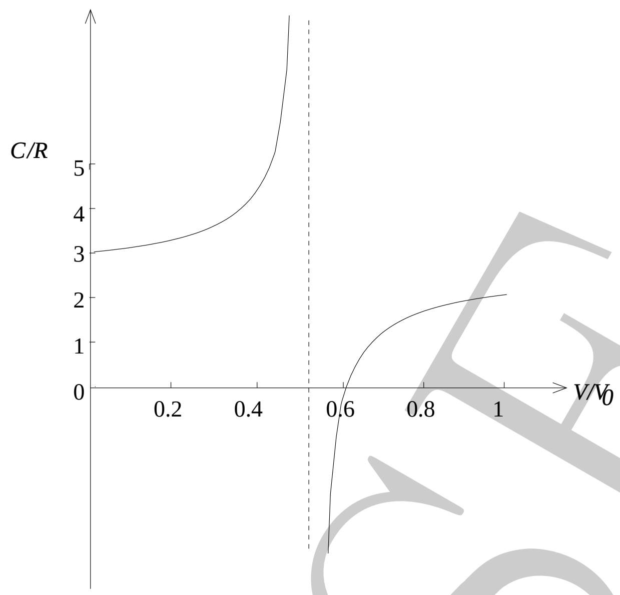


Figure 8: Problem 39 (j)

Chapter III

Detailed Solutions

1 INPhO-2006

1.

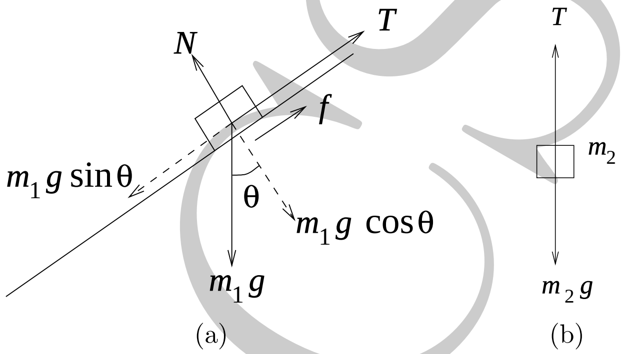


Figure 1: Free body diagrams assuming m_1 is going down: (a) For m_1. (b) For m_2.

(a) Let m_1 go down. Then from free body diagram for m_1,

$$m_1 \, g \, \sin\theta - f - T = m_1 \, a \tag{1}$$

where the frictional force $f = \mu N$, N being the normal reaction. Hence,

$$m_1 \, g \, \sin\theta - \mu N - T = m_1 \, a$$
$$N = m_1 g \cos\theta$$

Therefore
$$a = g \sin\theta - \mu g \cos\theta - \frac{T}{m_1} \tag{2}$$

From free body diagram for m_2,

$$T - m_2 g = m_2 a \tag{3}$$

Solving Eqs. (2) and (3),

$$a = \frac{g \, [\, m_1(\sin\theta - \mu\cos\theta) - m_2 \,]}{m_2 + m_1}$$

As $m_1 = m_2$,

$$a = \frac{g\,[\sin\theta - 1 - \mu\cos\theta\,]}{2}$$

We can see from the above equation that a can have only negative values. Substituting the values

$$a = -5.1 \;\; \text{m}\cdot\text{s}^{-2} < 0$$

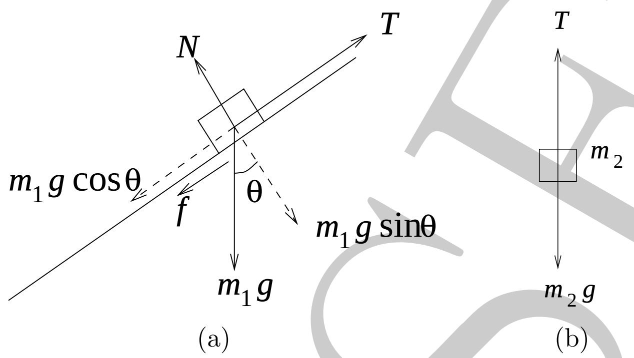


Figure 2: Free body diagrams assuming m_2 is going down: (a) For m_1. (b) For m_2.

Let m_2 go down. Then for m_2

$$m_2 g - T = m_2 a \tag{4}$$

For m_1

$$T - m_1 g\sin\theta - f = m_1\,a \tag{5}$$

Elementary algebraic manipulations yield

$$a = \frac{g\,[\,1 - \sin\theta - \mu\cos\theta\,]}{2}$$

Substituting the values

$$a = -0.1 \;\; \text{m}\cdot\text{s}^{-2} < 0$$

As seen from both the cases, the acceleration is negative, either assuming m_1 going down or m_2 going down. So we can conclude that the acceleration of the system is zero and **system would be stationary**.

(b) substituting $a = 0$ in Eq. (4),

$$T \cong 10 \text{ N}.$$

Since system is stationary, so frictional force would be static friction and a may have a positive value only if m_2 is going down and not up. By substituting the value of T and a in Eq. (5), we obtain

$$f_s = 5.2\,\text{N}.$$

This f_s is down the inclined plane. Why?

2.

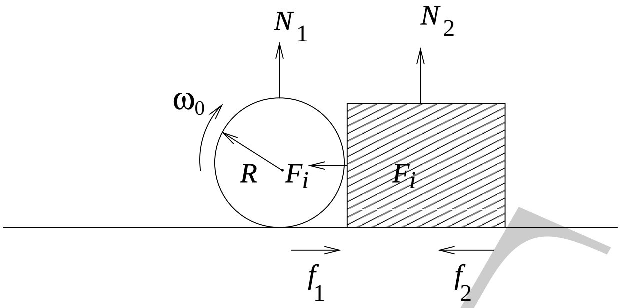


Figure 3: Free body diagram of the block disk system

Key points to remember are, when disk touches the block, block-disk system starts to slide. After some time t^*, disk starts to roll without slipping and after this time t^*, disk continues to roll without slipping and comes to a halt in total time t_{tot}.

(a) If a is the acceleration of the block-disk system, then by free body diagram(see Fig. (3)).

$$\begin{aligned} (2M+M)a &= f_1 - f_2 \\ 3Ma &= \mu(2M)g - \mu Mg \\ a &= \frac{\mu g}{3} \end{aligned} \tag{6}$$

(b) As disk rolls, it's angular velocity ω decreases. If α is the angular acceleration of the system, then

$$\begin{aligned} I\,\alpha &= -f_1 R \\ \frac{1}{2}(2M)R^2\alpha &= -\mu\,(2M)g\,R \\ \alpha &= \frac{-2\mu g}{R} \end{aligned} \tag{7}$$

At time t^*, disk starts to roll without slipping. So

$$v(t^*) = R\,\omega(t^*) \tag{8}$$

and equation of motion for rotational motion

$$\omega(t^*) = \omega_0 - \frac{2\,\mu\,g}{R}\,t^* \tag{9}$$

and equation of motion for translational motion

$$v(t^*) = 0 + \frac{\mu\,g}{3}\,t^* \tag{10}$$

By Eqs. (9) and (10)

$$t^* = \frac{3}{7}\cdot\frac{R\,\omega_0}{\mu\,g} \tag{11}$$

(c) After time $t > t^*$, velocity of block - disk system starts to decrease. In this condition of pure rolling, frictional force on disk $f_1' < \mu\,(2M)\,g$. Suppose angular acceleration is α' and linear acceleration is a', then

$$\alpha' = \frac{a'}{R}$$

$$I\,\alpha' = -f_1'\,R$$

$$\frac{1}{2}(2M)R^2\alpha' = -f_1'\,R$$

Hence

$$-f_1' = M\,a' \tag{12}$$

Now

$$(2M + M)a' = f_1' - f_2'$$

$$3M\,a' = -Ma' - \mu M g \Rightarrow a' = \frac{-\mu g}{4} \tag{13}$$

Suppose in time t from t^* the system comes to rest. Then

$$v(t) = v(t^*) + a't$$

Using Eqs. (10) and (13)

$$0 = \frac{\mu\,g}{3}\cdot\frac{3R\omega_0}{7\mu g} - \frac{\mu g}{4}\,t$$

$$= \frac{R\omega_0}{7} - \frac{\mu\,g\,t}{4}$$

$$t = \frac{R\omega_0}{\mu\,g}\,\frac{4}{7}$$

Thus total time

$$t_{tot} = t + t^*$$

$$t_{tot} = \frac{R\,\omega_0}{\mu\,g}$$

This is a surprisingly simple answer.

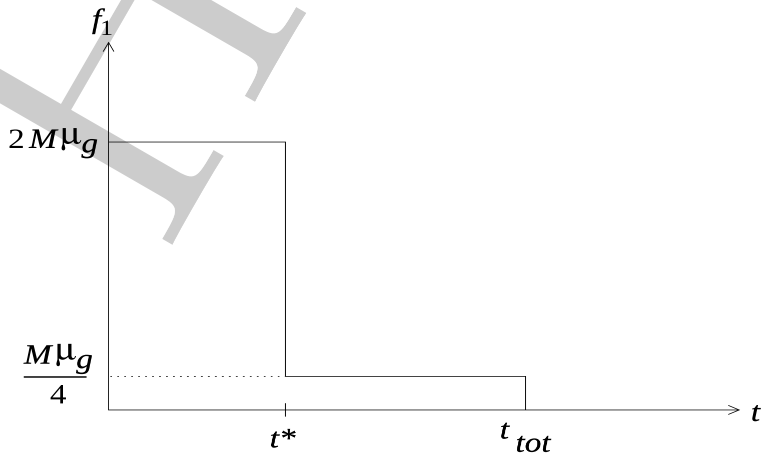


Figure 4: Plot of force of friction with time

Figure (4) gives the interesting plot of force of friction (f_1) with time. One can make an equivalent plot with angular velocity ω on the x-axis. There is a sudden drop in f, at $t = t* = 3r\omega_0/7\mu g$. This drop occurs when pure rolling sets in. It may remind the reader of a first order phase transition where f_1 plays the role of the internal energy and ω that of temperature.

3. (a) Efficiency of cycle is

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

In the cycle, energy is absorbed in isobaric process from a to b.
So $Q_{in} = C_p\,(T_b - T_a)$. Here C_p is the heat capacity at constant pressure.
Energy is released in isobaric process c to d.
So $Q_{out} = C_p\,(T_c - T_d)$. Hence

$$\eta = 1 - \frac{T_c - T_d}{T_b - T_a} \tag{14}$$

Another method exists for this solution. In this method

$$\eta = \frac{W}{Q_{in}}$$

and W is evaluated by integrating the area under the $P - V$ curve.

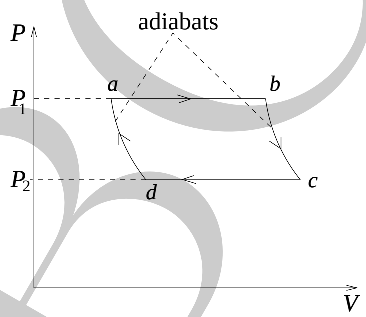


Figure 5: P-V diagram

(b) For adiabats b to c and d to a

$$PV^{\gamma} = \text{const.} \tag{15}$$

Equation of state gives

$$PV = nRT \Rightarrow \frac{PV}{T} = \text{const.}$$

Inserting this in Eq. (15)

$$T = \text{const.} \times P^{\frac{\gamma-1}{\gamma}} \tag{16}$$

Equation (16) gives

$$\frac{T_c}{T_d} = \frac{T_b}{T_a} \tag{17}$$

and

$$\frac{T_d}{T_a} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Equation (14) becomes

$$\eta = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

(c) For isobaric process, $V - T$ curve will be a straight line passing through the origin of the $V - T$ plot. Since $P_1 > P_2$ so for $a - b$, this line will be less steeper then $c - d$. By Eq. (15) and equation of state, we can get $T_b > T_c > T_a > T_d$. For convexity of curve from $a - d$ and $b - c$, we can analyze relation between $V$and T, that slope is inversely proportional to T. So on this basis we can draw $V - T$ diagram as shown in Fig. (6).

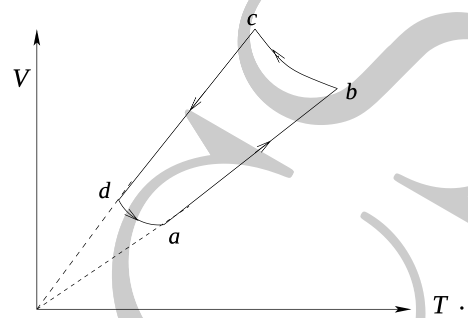


Figure 6: V-T diagram

(d) Carnot efficiency is defined by the highest temperature T_b and lowest temperature T_d :

$$\eta_c = 1 - \frac{T_d}{T_b}$$

Efficiency of concerned cycle (from Eqs. (14) and (17)) is

$$\eta = 1 - \frac{T_d}{T_a}$$

Hence

$$\frac{\eta_c}{\eta} = \frac{1 - \frac{T_d}{T_b}}{1 - \frac{T_d}{T_a}}$$

Since $T_b > T_a$, hence $\eta_c > \eta$ i.e. Carnot engine has higher efficiency than the concerned cycle.

4.

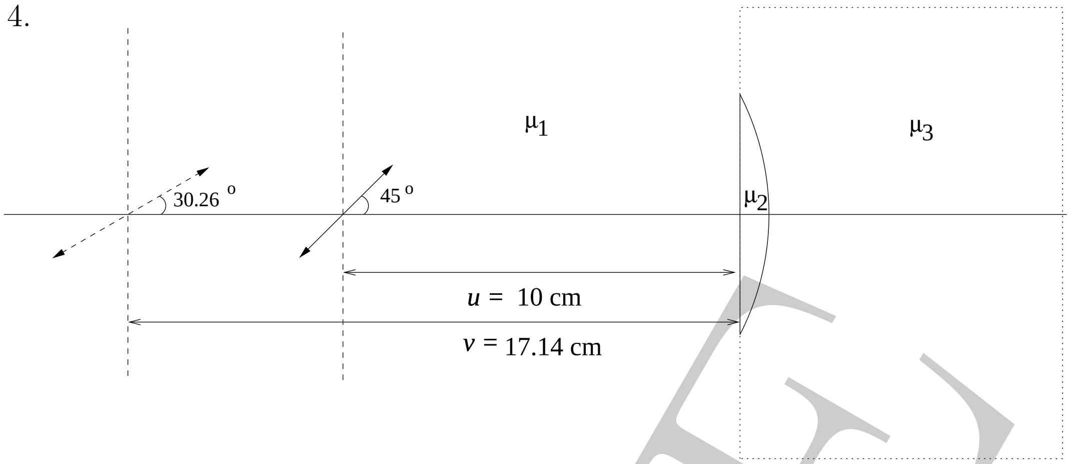


Figure 7: Problem 4

Given $\mu_1 = 1.0$, $\mu_2 = 1.5$, $\mu_3 = 1.2$, $u = -10\,\text{cm}$, $R = -10\,\text{cm}$.

General formula for thin lens,

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2}$$

Now $R_1 \longrightarrow \infty$, $R_2 = R$.

$$\text{Therefore} \qquad \frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_3 - \mu_2}{R} \tag{18}$$

$$\text{Implies} \qquad v = \frac{Ru\mu_3}{u(\mu_3 - \mu_2) + \mu_1 R} \tag{19}$$

$$= -17.14\,\text{cm}$$

Here the negative sign indicates that image is on the left side. Under small shift along the principal axis of the lens for the object, the image will also shift slightly along the same axis. So, for longitudinal magnification differentiate Eq. (19) with respect to v and get

$$\begin{aligned} \frac{dv}{du} &= \left(\frac{v}{u}\right)^2 \frac{\mu_1}{\mu_3} \\ &= 2.4 \end{aligned}$$

During the first refraction, ray travels from medium 1 (μ_1) to medium 2 (μ_2). Linear lateral magnification in first refraction

$$\frac{y'_2}{y_1} = \frac{\mu_1}{\mu_2} \frac{v'}{u} \tag{20}$$

For the second refraction

$$\frac{y_2}{y'_2} = \frac{\mu_2}{\mu_3} \frac{v}{v'} \tag{21}$$

Eq. (20) and Eq. (21) gives

$$\begin{aligned} \frac{y_2}{y_1} &= \frac{\mu_1}{\mu_3} \frac{v}{u} \\ &= 1.43 \end{aligned}$$

It is given that object's amplitude is $\sqrt{2}$ cm. Taking the projection of object along the principle axis and perpendicular to the axis,

$$\begin{aligned} du &= 1 \text{ cm (along the axis)} \\ \text{and} \quad y_1 &= 1 \text{ cm (normal to the axis)} \\ \Longrightarrow \quad dv &= 2.4 \text{ cm} \\ \text{and} \quad y_2 &= 1.4 \text{ cm.} \\ \text{Amplitude of the image} &= \sqrt{(dv)^2 + {y_2}^2} \\ &= 2.78\,\text{cm} \end{aligned}$$

Since lateral and longitudinal magnifications are positive, the phase difference between oscillating image and object will be zero. Only orientation of the axis along which oscillation takes place will be different.
Angle made by the virtual image with the principle axis

$$\begin{aligned} &= \tan^{-1}\left(\frac{y_2}{dv}\right) \\ &= \tan^{-1}\left(\frac{1.4}{2.4}\right) \\ &= 30.26^0 \end{aligned}$$

5.

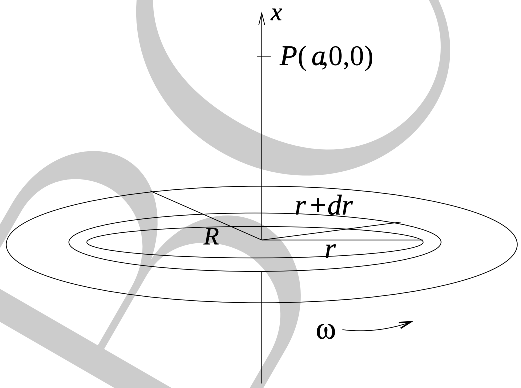


Figure 8: Problem 5

(a) Dipole moment due to a circular ring $(r, r+dr)$ (See Fig. (8))

$$\begin{aligned} d\mu' &= di\, A \\ \text{And} \qquad di &= \frac{dQ}{T} = \frac{2\pi\, r\, dr\, \sigma}{2\,\pi/\omega} \\ d\mu' &= \frac{2\pi r\, dr\, \omega\, \sigma}{2\pi}\,.\,\pi r^2 \end{aligned} \tag{22}$$

Then for circular disk

$$\begin{aligned} \mu' &= \pi\omega\sigma \int_0^R r^3\, dr \\ \mu' &= \frac{\pi\omega\sigma R^4}{4} \end{aligned}$$

Direction is in the positive x-direction.

(b) Magnetic field due to a circular ring $(r, r + dr)$

$$\begin{aligned} dB &= \frac{\mu_0}{2} \cdot \frac{di\, r^2}{(r^2 + a^2)^{3/2}} \\ B &= \frac{\mu_0}{2} \int_0^R \frac{di\, r^2}{(r^2 + a^2)^{3/2}} \end{aligned}$$

Using Eq. (22)

$$B = \frac{\mu_0\, \omega\, \sigma}{2} \int_0^R \frac{r^3}{(r^2 + a^2)^{3/2}}\, dr$$

Substitutions

$$\begin{aligned} r &= a \tan\theta \\ dr &= a \sec^2\theta\, d\theta \end{aligned}$$

and limit 0 to $\tan^{-1}\left(\frac{R}{a}\right)$

$$\begin{aligned} B &= \frac{\mu_0\, \omega\, \sigma\, a}{2} \int_0^{\tan^{-1}\left(\frac{R}{a}\right)} \tan^2\theta\, \sin\theta\, d\theta \\ &= \frac{\mu_0\, \omega\, \sigma\, a}{2} \int_0^{\tan^{-1}\left(\frac{R}{a}\right)} (\sec^2\theta - 1) \sin\theta\, d\theta \\ &= \frac{\mu_0\, \omega\, \sigma\, a}{2} \int_0^{\tan^{-1}\left(\frac{R}{a}\right)} (\sec^2\theta\, \sin\theta - \sin\theta)\, d\theta \end{aligned}$$

Integrating the first term by parts

$$B = \frac{\mu_0\, \omega\, \sigma\, a}{2} \left[\sin\theta\, \tan\theta + 2\cos\theta\right]_0^{\tan^{-1}\left(\frac{R}{a}\right)}$$

Applying the limits

$$B = \frac{\mu_0\, \omega\, \sigma}{2} \left[\frac{R^2 + 2\, a^2}{\sqrt{R^2 + a^2}} - 2\, a\right] \tag{23}$$

Direction is in the positive x-direction.

(c) **Method - I**
When $a >> R$ then for axial field due to dipole, we can use the formula

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \cdot \frac{2\mu'}{a^3} \\ &= \frac{\mu_0}{4\pi} \cdot \frac{2}{a^3} \cdot \frac{\pi\, \sigma\, \omega R^4}{4} \\ &= \frac{\mu_0\, \sigma\, \omega R^4}{8a^3} \end{aligned}$$

Direction is in the positive x-direction.
Method - II

From Eq. (23)

$$\begin{aligned} B &= \frac{\mu_0\,\omega\,\sigma}{2}\left[\frac{R^2+2\,a^2}{\sqrt{R^2+a^2}} - 2\,a\right] \\ &= \frac{\mu_0\,\omega\,\sigma}{2}\left[\frac{R^2+2\,a^2}{a}\left(1+\frac{R^2}{a^2}\right)^{-\frac{1}{2}} - 2a\right] \\ &= \frac{\mu_0\,\omega\,\sigma}{2}\left[\left(\frac{R^2}{a}+2a\right)\left(1-\frac{R^2}{2a^2}+\frac{3\,R^4}{8\,a^4}\right) - 2a\right] \\ &= \frac{\mu_0\,\omega\,\sigma R^4}{8a^3} \end{aligned} \tag{24}$$

Direction is in the positive x-direction.

(d) The force on dipole of dipole moment μ placed at a

$$F = \mu\frac{dB}{dx}$$

After differentiating Eq. (24) with respect to a,

$$F = -\frac{3\,\mu\,\mu_0\,\omega\,\sigma\,R^4}{8\,a^4}$$

Direction is dictated by $\vec{\mu}$.

6.

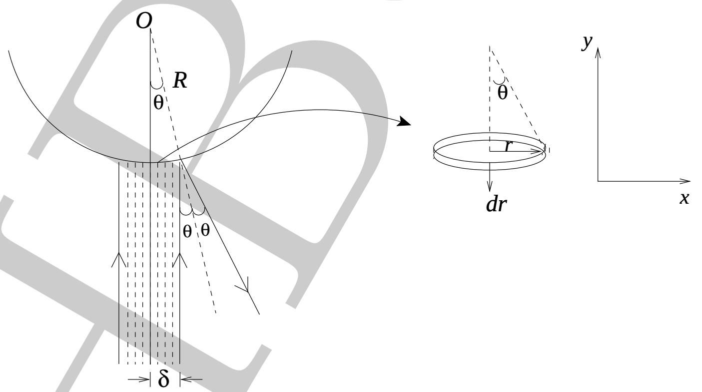


Figure 9: Problem 6

(a) Initial momentum of photon in the beam $\vec{p}_i = p\hat{\jmath}$.
Final momentum of photon in the beam $\vec{p}_f = p\,\sin 2\theta\,\hat{\imath} - p\,\cos 2\theta\,\hat{\jmath}$.
x component of momentum cancels by symmetry.
Net change in momentum $\Delta\vec{p} = -2p\cos^2\theta$.
The number of photons in the annular region r to $r+\Delta r$ per second $= n\;2\,\pi\,r\,dr$.
Magnitude of force on the annular region is $\Delta F = n2\pi\,r\,dr \times 2\,p\cos^2\theta$.

Total force on the sphere is (magnitude)

$$
\begin{aligned}
F &= \int_0^{\delta} 4\, n\pi\, p\, r\, \cos^2\theta\, dr \\
\sin\theta &= \frac{r}{R} \qquad\qquad \cos^2\theta = 1 - \frac{r^2}{R^2} \\
F &= 4\pi n p \left[\frac{\delta^2}{2} - \frac{\delta^4}{4\,R^2}\right] \qquad (25)
\end{aligned}
$$

Direction is upward so $\vec{F} = F\hat{\jmath}$.

(b) **Method - I**
Since $\delta << R$, drop δ^4/R^2 term in previous part.

$$F \approx 4\pi n p \frac{\delta^2}{2} \qquad (26)$$

If E (or $h\nu$) is the energy of one photon and P the power of the beam (1 kW), then

$$
\begin{aligned}
\frac{P}{E} &= n\pi\delta^2 \qquad (27) \\
\text{Also} \qquad E &= pc \qquad (28)
\end{aligned}
$$

Using Eqs. (27) and (28) in Eq. (26)

$$F = 2\frac{P}{c}$$

For levitation

$$
\begin{aligned}
F &= mg \\
m &= \frac{2P}{gc} = 6.80 \times 10^{-7}\,\text{kg}.
\end{aligned}
$$

Method - II

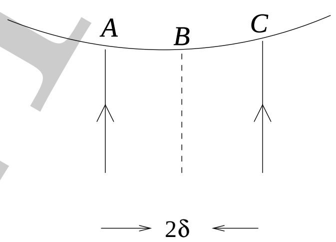


Figure 10: Problem 6 (b) method II

Alternatively one can obtain this without part(a).
Area $= \pi\,\delta^2$.
Since $\delta << R$, we can assume that A-B-C is flat.
Change in momentum per photon $= 2p$.

No. of photons per unit area per sec. $= n$
Therefore

$$\begin{aligned} F &= (n\,\pi\,\delta^2)\,2\,p \\ &= 2\,\pi\,n\,p\,\delta^2 \end{aligned}$$

This is the same as Eq. (26) and we proceed as in the previous part.

Although the answer to the question ends here, we draw your attention to the following. Let us estimate the size of this 'Al' sphere.

$$m = \frac{4\pi}{3} r^3 \rho$$

$$\begin{aligned} r^3 &= \frac{3m}{4\pi\rho} \\ &= \frac{6.8 \times 10^{-7}}{4 \times 2.7 \times 10^3} \times \frac{3}{3.14} \\ &= \frac{6.8 \times 3}{4 \times 2.7 \times 3.14} \times 10^{-10} \\ r &= 10^{-3} \times \left(\frac{6.8 \times 0.3}{4 \times 2.7 \times 3.14} \right)^{1/3} \\ &= 4 \times 10^{-4} \text{ m} = 0.4 \text{ mm}. \end{aligned}$$

Note that focussing a laser beam to a spot size $r/10 = 4 \times 10^{-5}$ m is easy. In this connection, recall that the "minimum" spot size is dictated by the wavelength of the beam. Laser levitations of polystyrene beads have been demonstrated. Optical tweezers is also a possibility. Thus it is possible to realise the experiment.

7. (a) Using formula

$$E_{n_2} - E_{n_1} = 13.6\,Z^2 \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2} \right) \text{ eV}$$

For Li^{++}, Z=3, and transition from $n_2 = 4$ to $n_1 = 3$.

$$E_4 - E_3 = \triangle E = h\nu = h\frac{c}{\lambda} = 13.6 \times 9 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) \text{ eV}$$

Putting the values of h and c,

$$\lambda = 2089\,\text{Å}$$

This wavelength belongs to the ultraviolet region of the electromagnetic spectrum.

(b)

$$h\nu = \phi + (\text{K.E.})_{max}$$

Here ϕ is the work function of metal. Putting all the values,

$$(\text{K.E.})_{max} = 2.75\,\text{eV}$$

Now de Broglie wavelength

$$\lambda_{dB} = \frac{h}{\sqrt{2\,m_e\,(\text{K.E.})_{max}}}$$
$$\lambda_{dB} = 7.4\,\text{Å}$$

8. (a) Equating Energies, (or Pressures, or Forces) for a spherical "charged" cloud

$$\frac{3}{5}\frac{Q^2}{4\,\pi\,\epsilon_0\,R} = \frac{3}{5}\frac{G\,M^2}{R}$$
$$Q = Nq = N(-ey)$$
$$M = Nm_p$$
$$y^2 = \frac{{m_p}^2}{e^2}\,G\,4\pi\,\epsilon_0$$
$$y = 0.9 \times 10^{-18} \simeq 10^{-18}$$

Hence

$$y \geqslant 10^{-18}$$

(b) The force on the atom at a distance r from the centre is

$$F = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q\,q}{r^2}$$

$$m\,\frac{dv}{dt} = m\,\frac{dv}{dr}.\frac{dr}{dt} = m\,v\frac{dv}{dr} \qquad \text{(chain rule)}$$
$$Q = \frac{\eta}{m_p}\,\frac{4\,\pi\,r^3}{3}\,q$$
$$q = -e\,y$$
$$v^2 \propto r^2 \quad \text{or} \quad v \propto r$$

with constant of proportionality

$$H = \left[\frac{\eta}{3\,\epsilon_0\,{m_p}^2}e^2\,y^2\right]^{\frac{1}{2}}$$

$$H = \frac{ey}{m_p}\left[\frac{\eta}{3\,\epsilon_0}\right]^{\frac{1}{2}}$$

(c)

$$H = \frac{ey}{m_p}\left[\frac{\eta}{3\,\epsilon_0}\right]^{\frac{1}{2}}$$

Putting right values of η, ϵ_0, m_p, e and $y = 10^{-17}$ (one order of magnitude larger)

$$H = 1.8 \times 10^{-17}\text{s}^{-1}$$

(d) **Method - I**

$$\begin{aligned}\frac{dr}{dt} &= H\,r\\ r &= r_0\,e^{Ht}\\ V &= V_0\,e^{3Ht}\end{aligned}$$

Method - II

$$\begin{aligned}\frac{dV}{dt} &= 3.\frac{4\pi\,r^2}{3}\frac{dr}{dt}\\ &= 3\,\frac{4\,\pi\,r^3}{3}\,H\\ \frac{dV}{V} &= 3\,H\,dt\\ V &= V_0\,e^{3Ht}\end{aligned}$$

(e) Constant H obtained in part (c) is physically similar to Hubble's constant. Observed value of Hubble's constant is $= 2.3\times10^{-18}\mathrm{s}^{-1}$, while obtained H is $= 1.8\times10^{-17}\mathrm{s}^{-1}$. Further, experiments do not indicate a difference in the magnitudes of the electron and proton charge. Some theories regarding the nature of the fundamental forces and elementary particles also do not point to a difference.

"This is not right. It is so bad, it is not even wrong!"
-Wolfgang Ernst Pauli (25 Apr.1900-15 Dec.1958) on the solution to a problem provided by a colleague.

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2 INPhO-2007

1. (a)

$$\text{Given} \quad r = at, \ \theta = \omega t \tag{29}$$

$$\text{Now} \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\text{Hence} \quad \vec{p} = m(a\hat{r} + r\omega\hat{\theta})$$

$$\vec{F} = m\vec{a}$$

$$= m[(\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}]$$

$$= m(-r\omega^2\hat{r} + 2a\omega\hat{\theta})$$

(b)

$$\int F_r dr = -m\omega^2 \int_0^r r dr = \frac{-m\omega^2 r^2}{2} \tag{30}$$

From Eq. (29)

$$\theta = \frac{r\omega}{a}$$

$$\int F_\theta \, r \, d\theta = 2m\omega^2 \frac{r^2}{2} = m\omega^2 r^2 \tag{31}$$

Adding Eqs. (30) and (31)

$$\Delta W = \frac{m\omega^2 r^2}{2}$$

We can also obtain the above result by employing the work-energy theorem.

(c) Trajectory will be a spiral.

2. (a) From Fig. (11)

$$v_y = v \sin\theta \tag{32}$$

Note that the surface is rough and there is frictional force along the x-direction. Hence elastic collision does not constrain the velocity along the x-direction. It implies that the y-component of the velocity $v\sin\theta$ changes only in sign. From Newton's second law along vertical y-direction, change in momentum is given by the linear impulse, which yields:

$$2mv\sin\theta = \int N \, dt \tag{33}$$

From Newton's second law along x-direction, we have

$$mv\cos\theta - \mu \int N \, dt = mv_x \tag{34}$$

Inserting Eq. (33) in Eq. (34) we obtain

$$v_x = v(\cos(\theta) - 2\mu\sin(\theta))$$

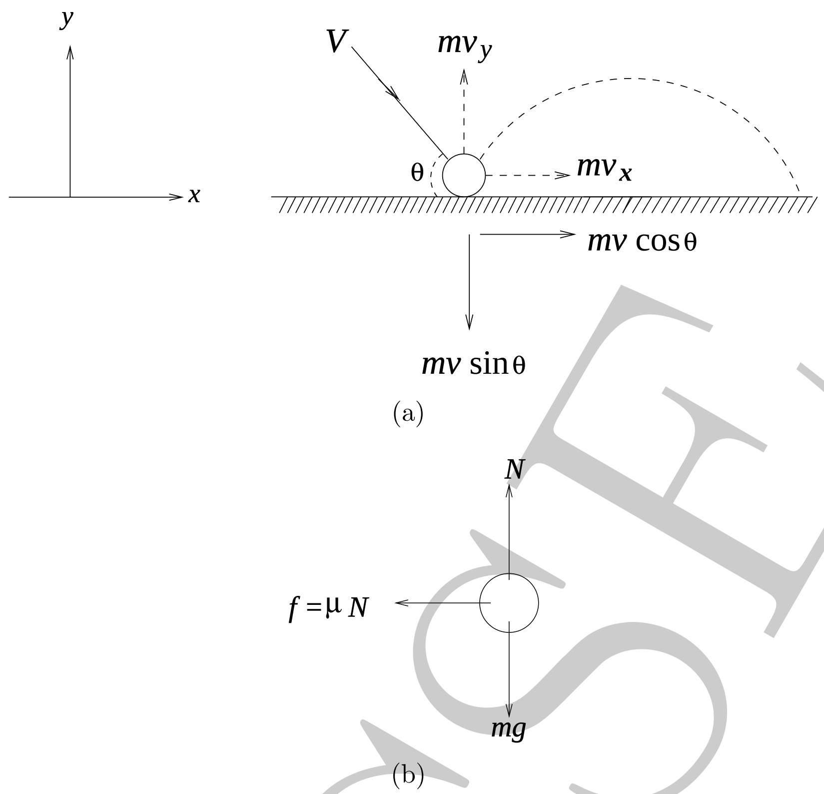


Figure 11: (a) Problem 2. (b) Free body diagram of the ball in contact with the floor.

$$\text{Range} = v_x \times \text{ time of flight} = v_x \times \frac{2v_y}{g}$$

$$R(\theta) = \frac{2v^2}{g}\, f(\theta)$$

where

$$f(\theta) = \sin\theta(\cos\theta - 2\mu\sin\theta)$$

To maximize R, set

$$f'(\theta_m) = 0$$

which yields

$$\theta_m \;=\; \frac{1}{2}\cot^{-1}(2\mu)$$

(b) Possible range of θ_m:

$$\theta_m \in]0,\, \pi/4[$$

3. See Fig. (12).

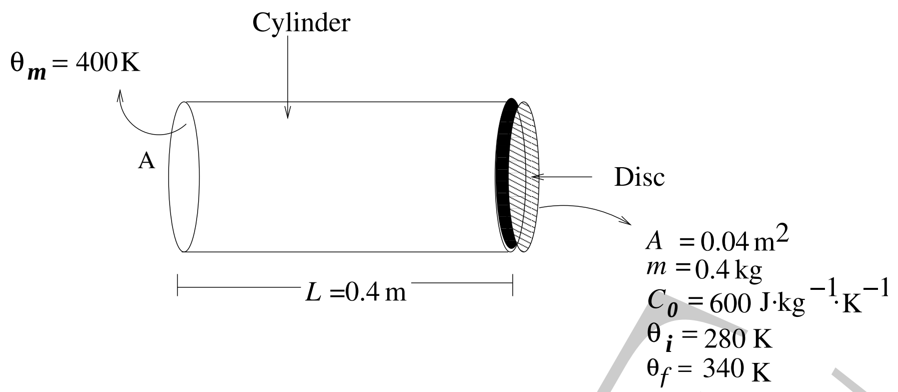


Figure 12: Problem 3

(a) By Fourier's law of heat conduction

$$\frac{dQ}{dt} = \frac{KA(\theta_m - \theta)}{L} \tag{35}$$

By calorimetry

$$\frac{dQ}{dt} = mC(\theta)\frac{d\theta}{dt} \tag{36}$$

By Eqs. (35) and (36)

$$\frac{1 - \alpha(\theta_i - \theta_m)}{\theta_m - \theta}\, d\theta - \alpha\, d\theta = \frac{KA}{mC_0\, L}\, dt$$

Integrating and noting that $mLC_0/KA = 240$ s

$$(1 - \alpha(\theta_i - \theta_m))\, \ln\left(\frac{\theta_m - \theta_i}{\theta_m - \theta_f}\right) - \alpha(\theta_f - \theta_i) = \frac{t}{240} \tag{37}$$

Inserting the values $t \approx 222$ s.

(b) If $\alpha = 0$, then from Eq. (37)

$$\begin{aligned} \frac{t}{240} &= \ln\left(\frac{\theta_m - \theta_i}{\theta_m - \theta_f}\right) \\ t &= 166 \text{ s} \end{aligned}$$

(c) Process (3a) takes more time since as the disk heats up, its specific heat also increases and more heat is required to effect a further rise in temperature. Note

$$C(\theta_i) = 600 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$
$$C(\theta_f) = 960 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$

4.

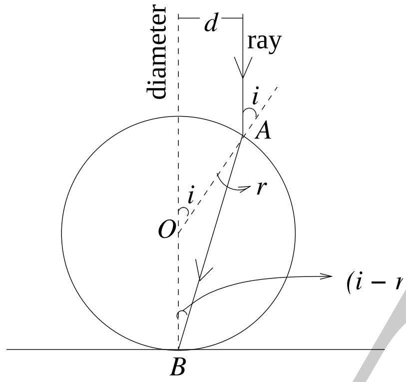


Figure 13: Problem 4

(a) From sine rule for $\triangle OAB$ (see Fig. (13))

$$\frac{OA}{\sin(i-r)} = \frac{OB}{\sin r}$$

$$\begin{aligned} OA = OB &= R \\ \therefore \sin r &= \sin(i-r) \\ &= \sin i \cos r - \cos i \sin r \end{aligned}$$

Using Snell's law for refraction

$$\sin i = n \sin r$$

$$\text{Hence } \frac{1}{n} = \frac{\sqrt{n^2 - \sin^2 i}}{n} - \frac{\sqrt{1-\sin^2 i}}{n}$$

From Fig. (13), $\sin i = d/R$.
Solving above equations

$$d^2 = R^2\left(1 - \frac{(n^2-2)^2}{4}\right)$$

(b) d ranges from 0 to R.
If $d \to 0$.

$$n = \pm 2 \text{ or } 0$$

Only physically allowed value is $n = +2$.
If $d \to R$.

$$n = \pm\sqrt{2}$$

Only allowed value is $+\sqrt{2}$.
Hence the allowed range of n is

$$n \in]\sqrt{2}, 2[$$

5. (a) Using Gauss law, the electric field is

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$$

(b) Integrating along a radial line, we get the electric potential

$$V(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{a}\right)$$

where a is the distance of reference point from the line charge.

(c) Total potential at point $P \equiv (x, y)$ is (see Fig. (14))

$$V(P) = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_+}{r_-}\right)$$

Equipotential lines in $z = 0$ plane are given by the equation $V(P) = V_0$, some constant. Let $k = \exp(2\pi\epsilon_0 V_0/\lambda)$. Then equation for equipotential lines becomes

$$\begin{aligned} \frac{r_-}{r_+} &= k \\ \frac{(x+d)^2+y^2}{(x-d)^2+y^2} &= k^2 \\ \left(x - d\frac{k^2+1}{k^2-1}\right)^2 + y^2 &= \frac{4d^2k^2}{(k^2-1)^2} \end{aligned}$$

This is an equation of a circle with centre at $\left(d\frac{k^2+1}{k^2-1}, 0\right)$ and radius given by $\frac{2dk}{|k^2-1|}$.

(d) See Fig. (14)

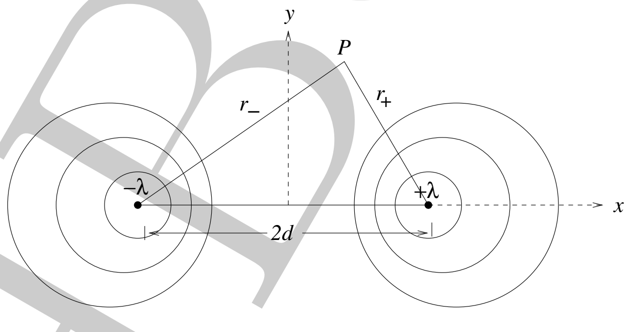


Figure 14: Problem 5 (d)

(e) Magnitude of electric force per unit length is given by

$$F_E = \frac{\lambda^2}{4\pi\epsilon_0 d}$$

Magnitude of magnetic force per unit length is given by

$$F_M = \frac{\mu_0\lambda^2 v^2}{4\pi d}$$

If these are equal then $v = 1/\sqrt{\epsilon_0\mu_0} = c$. This emphasizes the general observation that magnetic force is very small compared to the electrostatic force.

6.

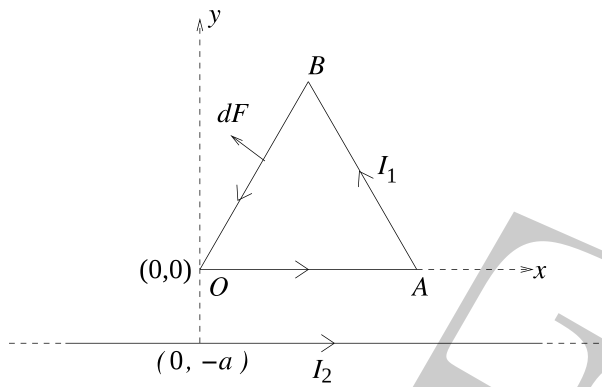


Figure 15: Problem 6 (a)

(a) See Fig. (15). Force F_1 on side OA is

$$F_1 = \frac{\mu_0 I_2 I_1 S}{2\pi a}$$

downwards, i.e. the negative y - direction.
Consider element dr at a distance r from O on OB. On both of the sides (OB and AB) same magnitude of force acts. The x components cancel and the y components add. Hence total force on both sides

$$F_{2y} = \frac{\mu_0 I_1 I_2}{2\pi} \int_0^S \frac{1}{a + \sqrt{3}\, r/2} dr$$

upwards, i.e. the positive y - direction.
By adding these forces, total force on the triangle is

$$F = \frac{\mu_0 I_1 I_2}{\sqrt{3}\pi} \left[\frac{\sqrt{3}S}{2a} - \ln\left(1 + \frac{\sqrt{3}S}{2a}\right) \right]$$

This net force will be downwards.

(b) See Fig. (16). The behaviour is quadratic for small S/a and almost linear for large S/a.

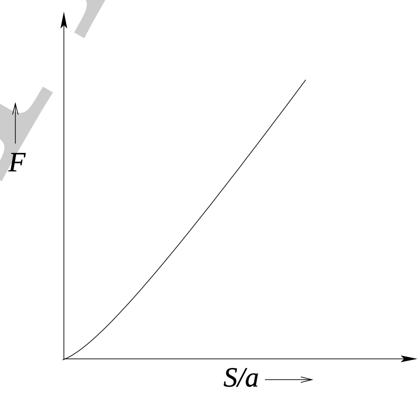


Figure 16: Problem 6 (b)

7. (a) Let n be the number of photons per second and P, the power of the source.
Change in momentum of one photon

$$= \frac{h}{\lambda}(\hat{\jmath} - \hat{\imath})$$

Rate of change of linear momentum of the beam is

$$\vec{F} = \frac{nh}{\lambda}(\hat{\jmath} - \hat{\imath})$$

$$= \frac{P}{c}(\hat{\jmath} - \hat{\imath}) \qquad \left(\because P = \frac{nhc}{\lambda}\right)$$

Inserting values

$$|\vec{F}| = 2.82 \times 10^{-7}\,\mathrm{N}$$

(b) Surface charge density σ of metal surface after time t with photoelectric efficiency η.

$$\begin{aligned} &= \frac{n}{A} e\, t\, \eta \\ &= \frac{P\lambda}{hcA} e\, t\, \eta \end{aligned}$$

Now $t = 10$ s and $\eta = 0.1$. Hence

$$\sigma = 7.27 \times 10^{2}\,\mathrm{C{\cdot}m^{-2}}$$

(c) Energy density after 10 s

$$\begin{aligned} &= \frac{1}{2}\epsilon_0 \left(\frac{\sigma}{\epsilon_0}\right)^2 \\ &= 2.93 \times 10^{16}\,\mathrm{J{\cdot}m^{-3}} \end{aligned}$$

(d) Energy of one photon

$$E = \frac{hc}{\lambda} = 2.06\,\mathrm{eV}$$

Given that work function

$$\phi = 1.9\,\mathrm{eV}$$

Then
K_{max} = E - ϕ = 0.16 eV
Hence the range of kinetic energy is from 0 to 0.16 eV.

8.

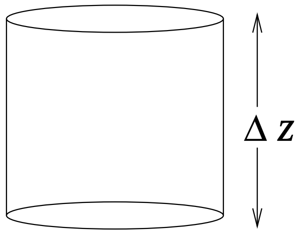


Figure 17: Problem 8

(a) From Fig. (17)

$$\Delta p = -\rho g \Delta z$$

Let m_a be the mass of one mole of air. Then by ideal gas law

$$p = \frac{\rho RT}{m_a}$$

Also, the equation for adiabatic process is $p^{\gamma-1} \propto T^{\gamma}$
From these equations,

$$\Delta T = -\frac{(\gamma-1)}{\gamma}\frac{m_a g}{R}\Delta z$$

The lapse rate equation is

$$\Gamma = -\frac{(\gamma-1)}{\gamma}\frac{m_a g}{R}$$

(b) For diatomic gas $\gamma = 7/5$ and $m_a = 2.9 \times 10^{-2}$ kg·mol^{-1}.
Hence $\Gamma = -0.01$ K·m^{-1} (i.e. 1^0 C decrease for every 100 m).

(c) Using equations of part (8a)

$$\frac{dp}{p} = -\frac{m_a g}{R}\frac{dz}{(T_0 - \Gamma\, z)}$$

On integrating

$$p = p_0\left(\frac{T_0 - \Gamma\, z}{T_0}\right)^{m_a g/R\Gamma}$$

There are two interesting aspects about the exponent of the above equation. The first is a rare occurance of "Force" related dimension in gases, namely $R\Gamma$. The second is its numerical value $m_a g/R\Gamma \simeq 3$.
For reasonable heights (e.g. $z = 1$ km)

$$p(z) \simeq p_0\left[1 - \frac{m_a g z}{RT_0}\right]$$

(d) Inserting given values in the above equation, the height of the atmosphere is approximately 30 km.

9. (a) The magnetic field at the centre of the field coil is

$$B = \frac{\mu_0}{2}\frac{N_f}{R_f}I(t)$$

(b) Flux on the pick-up coil

$$\phi = \pi {R_p}^2\, B$$

Induced emf in N_p turns

$$\begin{aligned} \varepsilon &= -N_p\frac{d\phi}{dt} \\ |\varepsilon| &= \frac{\mu_0}{2}\frac{N_p N_f}{R_f}\pi {R_p}^2\frac{dI(t)}{dt} \end{aligned}$$

(c) For maximum ε

$$\varepsilon_0 = \frac{\mu_0}{2}\frac{N_p N_f}{R_f}\pi {R_p}^2\, I_0\, \omega$$
$$\therefore \quad N_p N_f = 645 \tag{38}$$

(d) Mutual inductance on field coil due to pick up coil is equal to mutual inductance on pick up coil due to field coil. Hence mutual inductance

$$M = \frac{\mu_0}{2}\frac{N_p N_f}{R_f}\pi {R_p}^2 = 1.59 \times 10^{-5}\,\mathrm{H}$$

(e) The length of wire used is

$$L = 2\pi R_f N_f + 2\pi R_p N_p$$

To optimize it

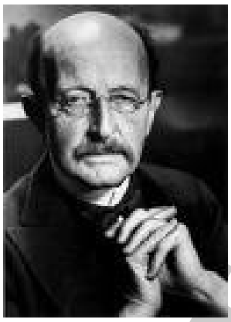
$$\frac{dL}{dN_f} = 0$$

Eq. (38) yields

$$N_f = 18\,\text{turns},\ \ N_p = 36\ \text{turns}$$

(f) Induced emf in case of

i. Iron : will increase.
ii. Wood : no appreciable change.
iii. Copper: decrease.



Max Planck (23 Apr.1858-4 Oct.1947) : Max (Karl Ernst Ludwig) Planck was a German theoretical physicist. He studied at Munich and Berlin, where he studied under Helmholtz, Clausius and Kirchoff and subsequently joined the faculty. He became professor of theoretical physics (1889-1926). His work on the law of thermodynamics and the distribution of radiation from a black body led him to abandon classical Newtonian principles and introduce the quantum theory (1900). For this he was awarded the Nobel Prize for Physics in 1918. This assumes that energy is not infinitely divisible, but ultimately exists as discrete amounts he called quanta (Latin, "how much"). Further, the energy carried by a quantum depends in direct proportion to the frequency of its source radiation.
The work leading to the "lucky" blackbody radiation formula was described by Planck in his Nobel Prize acceptance speech (1920): **"But even if the radiation formula proved to be perfectly correct, it would after all have been only an interpolation formula found by lucky guess-work and thus, would have left us rather unsatisfied. I therefore strived from the day of its discovery, to give it a real physical interpretation and this led me to consider the relations between entropy and probability according to Boltzmann's ideas. After some weeks of the most intense work of my life, light began to appear to me and unexpected views revealed themselves in the distance."** [See also the Foreword]

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1. Note that
A: has dimensions of energy.
B: has dimensions of length.
C: is dimensionless.
Now the Bohr radius will have a combination of B and C.

$$r_n = \frac{n^2}{2\pi}\frac{B}{C}$$

The energy will have a combination of A and C.

$$E_n = -\frac{1}{2n^2} AC^2$$

The Rydberg constant (of dimensions length inverse) will have a combination of B^{-1} and C.

$$R = \frac{C^2}{2B}$$

Note: This problem can be done in number of ways.

2.

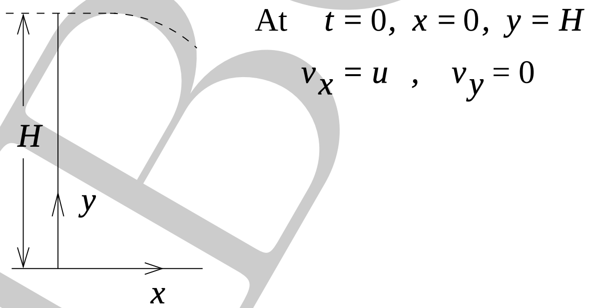

Figure 18: Problem 2 (a)

(a) See Fig.(18)

$$\frac{dv_x}{dt} = -cv_x$$

$$x = \frac{u}{c}(1 - e^{-ct}) \tag{39}$$

$$\frac{d\,v_y}{dt} = -g - cv_y$$

$$\frac{dy}{dt} = -\frac{g}{c}\left(e^{-ct} - 1\right)$$

$$\text{and} \qquad y = H + \frac{g}{c^2} - \frac{g}{c}\left[\frac{1}{c}\,e^{-ct} + t\right] \tag{40}$$

(b) From Eq. (39)

$$t = -\frac{1}{c}\ln\left(1 - \frac{xc}{u}\right)$$

Substituting in Eq. (40),

$$y = H + \frac{g}{c^2} - \frac{g}{c}\left[\frac{1}{c}e^{\ln\left(1-\frac{xc}{u}\right)} - \frac{1}{c}\ln\left(1 - \frac{xc}{u}\right)\right]$$

$$y = H + \frac{g}{c^2} - \frac{g}{c^2}\left(1 - \frac{xc}{u}\right) + \frac{g}{c^2}\ln\left(1 - \frac{xc}{u}\right)$$

$$y = H + \frac{gx}{cu} + \frac{g}{c^2}\ln\left(1 - \frac{xc}{u}\right)$$

If c is small and the range is limited, e.g. $\frac{xc}{u} << 1$ then

$$y = H + \frac{gx}{cu} + \frac{g}{c^2}\left[-\frac{xc}{u} - \frac{x^2\,c^2}{2\,u^2} - \frac{x^3\,c^3}{3\,u^3} + \cdots\cdots\right] \tag{41}$$

$$y = H - \frac{g\,x^2}{2\,u^2} - \frac{g\,x^3\,c}{3\,u^3}$$

(c) The trajectory is foreshortened (see Fig. (19)).
Note $c = 0$ is without air resistance
and $c \neq 0$ is with air resistance.

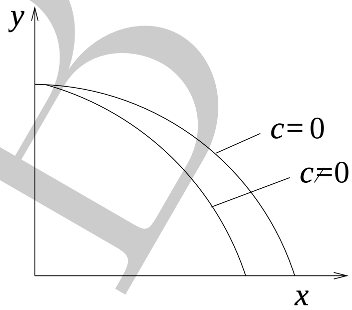

Figure 19: Problem 2 (c)

(d) When $y = 0$, from Eq. (40)

$$H + \frac{g}{c^2}\left[1 - e^{-ct} - ct\right] = 0$$

For $H = 500$ m, $c = 0.05\ \mathrm{s^{-1}}$, $g = 10\ \mathrm{m{\cdot}s^{-2}}$

$$t = 10 + 1.1 = 11.1\ \mathrm{s}$$

The above answer is obtained by iterative analysis. You may verify that the approximation in Eq. (41) is valid.

3. (a) Free Body Diagram, see Fig. (20)

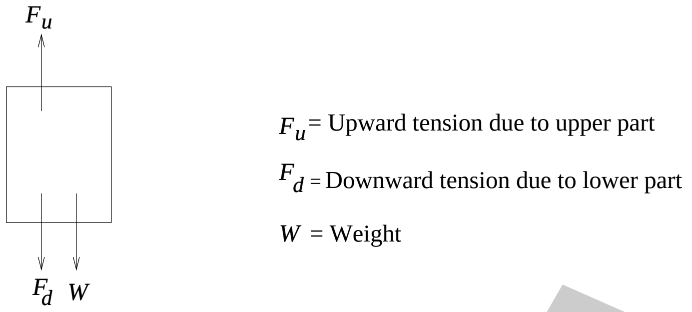

Figure 20: Problem 3 (a)

(b)

$$F_U - F_D = W - F_C$$

$$A\,(dT) = \frac{GM\rho\,A\,dr}{r^2} - \omega^2\,r\,\rho\,A\,dr$$

$$\frac{dT}{dr} = GM\rho\left[\frac{1}{r^2} - \frac{r}{R_g^3}\right] \quad \text{(using Kepler's third law)} \tag{42}$$

(c) Integrating Eq. (42)

$$\int_a^b dT = \int GM\rho\left(\frac{1}{r^2} - \frac{r}{R_g^3}\right) dr$$

$$T_b - T_a = GM\rho\left[-\frac{1}{r} - \frac{r^2}{2\,R_g^3}\right]\Bigg|_a^b \tag{43}$$

For $r \to R_g$ and $T(R) = 0$

$$T(R_g) = GM\rho\left[-\frac{3}{2\,R_g} + \frac{R^2}{2\,R_g^3} + \frac{1}{R}\right] \tag{44}$$

For $r \to R_g$ and $T(H) = 0$

$$T(R_g) = GM\rho\left[-\frac{3}{2\,R_g} + \frac{H^2}{2\,R_g^3} + \frac{1}{H}\right] \tag{45}$$

Equating Eqs. (44) and (45) we obtain

$$\begin{aligned} H &= \frac{R}{2}\left[\sqrt{1 + \frac{8\,R_g^3}{R^3}} - 1\right] \\ &= 1.51 \times 10^5 \;\; \text{km} \end{aligned}$$

(d) See Fig. (21)

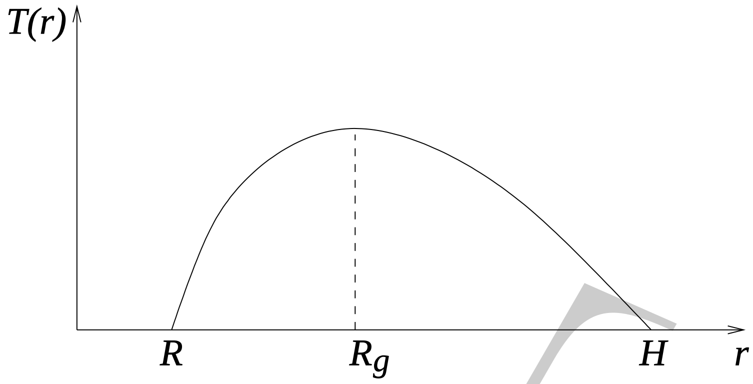

Figure 21: Problem 3 (d)

(e) Maximum stress will be at $r = R_g$
Using Eq. (44)

$$T(R_g) = 379 \;\; \text{GPa}$$

Steel has tensile strength 6.37 GPa which is less than 379 GPa. Hence it will not be feasible.

4.

$$T_0 = \frac{T_1 + T_2}{2}, \;\; \Delta = T_2 - T_1, \;\; \delta = T'' - T'$$

$T_1 = 270\text{K}$, $T_2 = 298\text{K}$

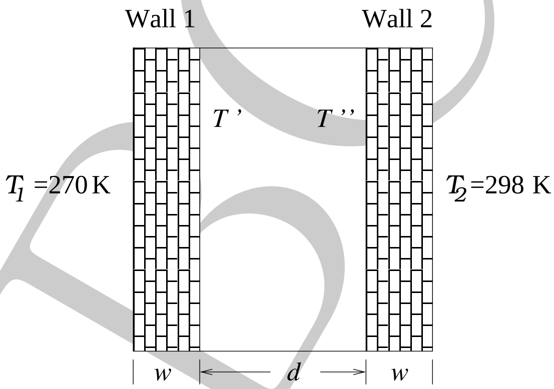

Figure 22: Problem 4

(a) For wall 1, $q_w = \frac{k_w}{w}(T_2 - T'')$

For wall 2, $q_w = \frac{k_w}{w}(T' - T_1)$

(b)

$$\Delta - \delta = T_2 - T'' + T' - T_1$$

For stationary process,

$$T_2 - T'' = T' - T_1 \tag{46}$$

$$\Delta - \delta = 2(T_2 - T'')$$

Hence,

$$q_w = \frac{k_w}{w} \; \frac{(\Delta - \delta)}{2}$$

(c)

$$E_1 = \epsilon\sigma T'^4 + (1-\epsilon)E_2$$

and

$$E_2 = \epsilon\sigma T''^4 + (1-\epsilon)E_1$$

$$q_r = E_2 - E_1$$
$$= \frac{\epsilon\,\sigma}{(2-\epsilon)}\,(T''^4 - T'^4)$$

(d) Using Eq. (46) and given set of equations,

$$T'' = T_0 + \frac{\delta}{2}, \quad T' = T_0 - \frac{\delta}{2} \tag{47}$$

also,

$$T''^4 - T'^4 = (T''^2 - T'^2)(T''^2 + T'^2)$$
$$= 2\delta\,T_0\,[(T'' + T')^2 - 2T''\,T']$$

putting values from Eq. (47)

$$= 4\delta\,T_0^3\left[1 + \left(\frac{\delta}{2\,T_0}\right)^2\right]$$

again, $\quad q_r = q_w$

$$\frac{\epsilon\,\sigma}{(2-\epsilon)}\,(T''^4 - T'^4) = \frac{k_w}{w}\,\frac{(\Delta-\delta)}{2}$$

since, $\quad (\delta^2 << T_0^2)$

$$\frac{\epsilon\,\sigma}{(2-\epsilon)}\,4\,\delta\,T_0^3 = \frac{k_w}{w}\,\frac{(\Delta-\delta)}{2}$$

$$1 - \frac{\delta}{\Delta} = 8\,c\,T_0^3\,\frac{\delta}{\Delta}$$

where,

$$c = \frac{\sigma\,\epsilon\,w}{k_w\,(2-\epsilon)}$$

$$\delta = \frac{\Delta}{1 + 8cT_0^3}$$

Now,

$$q_r = q_w = \frac{k_w}{w}\,\frac{(\Delta-\delta)}{2} \;=\; \frac{k_w}{w}\left(\frac{\Delta\,4c\,T_0^3}{1 + 8c\,T_0^3}\right)$$

(e)

$$c = 6.44\times10^{-10},\ w = 0.01\ \text{m},\ k_w = 0.72\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$$

$$T_0 = 284\ \text{K},\ \Delta = 28\,\text{K}$$

$$q_r = 107.22\ \text{W}\cdot\text{m}^{-2}$$

(f)

$$q_{cv} = \frac{N_u\,K_a}{d}\,(T'' - T') = \frac{N_u\,K_a}{d}\,\delta$$

$$q_{cv} = q_w$$

gives,

$$\delta = \frac{\Delta}{1 + \dfrac{2\,w\,k_a\,N_u}{k_w\,d}}$$

$$q_{cv} = \frac{N_u\,k_a\,\Delta\,k_w}{k_w\,d + 2\,w\,k_a\,N_u}$$

(g) Ignoring $2wk_a\,N_u$
$q_{cv} \simeq 46.5\ \ \mathrm{W{\cdot}m^{-2}}$

(h)

$$q_{cd} = \frac{k_s}{d}(T'' - T') = \frac{k_s\delta}{d}$$

$$q_{cd} = q_w$$

From part (b),

$$\frac{k_w}{2\,w}(\Delta - \delta) = \frac{k_s\,\delta}{d}$$

which gives,

$$\delta = \frac{k_w\,\Delta\,d}{2\,wk_s + k_w\,d}$$

Hence,

$$q_{cd} = \frac{k_s\,k_w\,\Delta}{2\,wk_s + k_w\,d}$$

(i) $k_s = 0.05\ \mathrm{W{\cdot}m^{-1}K^{-1}}$, $k_w = 0.72\ \mathrm{W{\cdot}m^{-2}}$
$w = 0.01$ m, $d = 0.1$ m
$q_{cd} = 13.8\ \mathrm{W{\cdot}m^{-2}}$

(j) Since,

$$q_{cd}\ <\ q_{cv}\ <\ q_r$$

Hence sheathing material is best for insulation.

5. See the brief solution.

6. (a) The force on one capacitor plate due to the other is

$$F_e = \frac{Q_1^2}{2A\epsilon_0}$$

$$F_e = \frac{C_1^2 V_0^2 \cos^2(2\pi f t)}{2\pi a^2 \epsilon_0}$$

Time - averaged force is

$$\langle F_e \rangle = \frac{C_1^2\,V_0^2}{4\,\pi\epsilon_0\,a^2} \tag{48}$$

(b) Charge on Capacitor C_2: $Q_2 = C_2 V_0 \cos(2\pi f t)$

$$i_2 = -C_2 V_0 \, 2\pi f \sin(2\pi f \, t)$$

(note: wire has negligible resistance.)
Force on one ring due to the other

$$F_m = i_2 l B$$

Hence,

$$F_m = \frac{\mu_0 \, b}{h}(-C_2 V_0 2\pi f \sin(2\pi f t))^2$$

Time- averaged force is

$$\langle F_m \rangle = \frac{\mu_0 \, b}{2\,h} C_2^2 V_0^2 (2\pi f)^2 \tag{49}$$

(c) Equating Eqs. (48) and (49)

$$\langle F_e \rangle = \langle F_m \rangle$$

$$\text{and noting } \; c^2 = \frac{1}{\mu_0 \, \epsilon_0}$$

We obtain

$$c = (2\pi)^{3/2} \, a \left(\frac{b}{h}\right)^{1/2} \frac{C_2}{C_1} f \tag{50}$$

(d) From Eq. (50) and given constants.

$$c = 2.99 \times 10^8 \text{ m·s}^{-1}$$

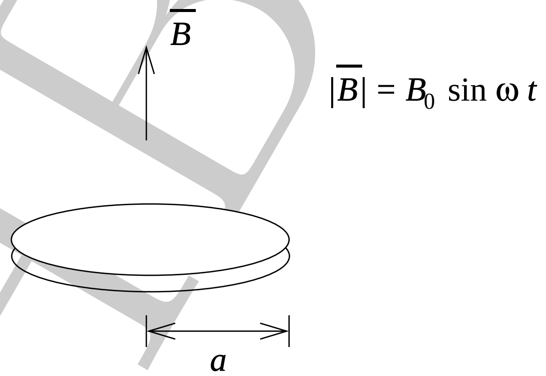

Figure 23: Problem 7

7. (a) Flux : $\phi = B_0 \sin(\omega t) \, \pi a^2 \, N$

$$\varepsilon = -\frac{d\phi}{dt} = -N\pi a^2 B_0 \, \omega \cos \omega t$$

From Kirchhoff's Law,

$$i\,R + L\,\frac{di}{dt} = -N\pi a^2 B_0 \, \omega \cos \omega t \tag{51}$$

(b) Take $i = Rl(i_0 e^{j\omega t})$ $(j^2 = -1)$
Substituting the complex form in Eq. (51), we obtain,

$$i_0 = \frac{N\,\pi\,a^2 B_0\,\omega(R - j\omega L)}{R^2 + \omega^2\,L^2}$$

This implies,

$$i = \frac{N\,\pi\,a^2 B_0\,\omega(R\cos\omega t + \omega L\sin\omega t)}{R^2 + \omega^2\,L^2}$$

(c) The elemental force

$$d\overline{F} = i\,d\bar{l} \times \overline{B}$$

is directed radially in. Substituting,

$$\frac{dF}{dl} = -\frac{NB_0^2\,\pi a^2\omega}{R^2 + \omega^2 L^2}(R\,\sin\omega t\,\cos\omega t + \omega\,L\sin^2\omega t)$$

Time - averaged compressional force

$$\left.\frac{dF}{dl}\right|_{av} = -\frac{NB_0^2\,\pi a^2\omega^2 L}{2(R^2 + \omega^2 L^2)} \tag{52}$$

$$\left.\frac{dF}{dl}\right|_{osc} = -\frac{NB_0^2\,\pi a^2\omega}{2(R^2 + \omega^2 L^2)}(R\sin 2\omega t - \omega L\cos 2\omega t) \tag{53}$$

Net force on ring is zero by symmetry.

(d) From Eq. (52)

$$\begin{aligned}\left.\frac{dF}{dl}\right|_{av} &= -\frac{\pi(10^{-2})\,10^6\,10^{-1}}{2(10^2 + 10^4)} \\ &\simeq -\frac{\pi}{2}10^{-1}N \\ &= 1.55\ \text{N}\cdot\text{m}^{-1}\end{aligned}$$

(e) i. From Eq. (53) the oscillating force has a frequency of 2ω and hence the frequency of the sound is 120 Hz.

ii. The inclusion of the capacitor will result in
$\omega L \to \omega L - \dfrac{1}{\omega C}$
$\therefore$ the compressional force is lessened and may even become negative, i.e. tensile and outward.

(8 June 1936 -)

"In the fall of 1970 Ben Widom asked me to address his statistical mechanics seminar on the renormalization group. He was particularly interested because Di Castro and Jona-Lasinio had proposed applying the field theoretic renormalization group formalism to critical phenomena, but no one in Widom's group could understand Di Castro and Jona-Lasinio's paper. In the course of lecturing on the general ideas of fixed points and the like I realized I would have to provide a computable example, even if it was not accurate or reliable. I applied the phase space cell analysis to the Landau-Ginzburg model of the critical point and tried to simplify it to the point of a calculable equation, making no demands for accuracy but simply trying to preserve the essence of the phase space cell picture. The result was a recursion formula in the form of a nonlinear integral transformation on a function of one variable, which I was able to solve by iterating the transformation on a computer. I was able to compute numbers for exponents from the recursion formula at the same time that I could show (at least in part) that it had a fixed point and that the scaling theory of critical phenomena of Widom et *al.* followed from the fixed point formalism. Two papers of 1971 on the renormalization group presented this work."

Kenneth Wilson, Nobel Laureate, 1982 on how a Nobel prize winning work was born from an effort to cook up a simple example. [See also the Foreword]

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(39).

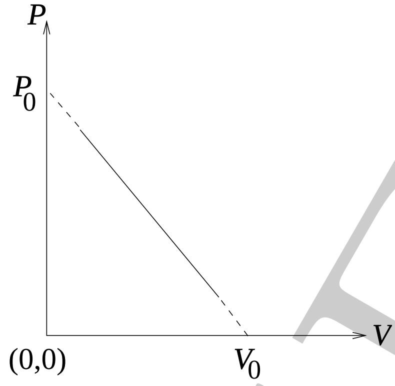


Figure 24: Problem 39

(a) Since the P-V plot is linear (see Fig. (24))

$$P = mV + C$$

Note that at $P = 0$, $V = V_0$. Hence

$$C = -mV_0$$

$$m = \frac{-P_0}{V_0}$$

$$P = \frac{-P_0}{V_0}(V - V_0) \tag{54}$$

Rewriting we obtain

$$\frac{P}{P_0} + \frac{V}{V_0} = 1 \qquad (P < P_0, V < V_0) \tag{55}$$

(b) The ideal gas law ($PV = RT$) for one mole of the gas implies

$$P = \frac{RT}{V}$$

Using Eq.(55),

$$\frac{RT}{V} = -\frac{P_0}{V_0}(V - V_0)$$

Rewriting,

$$T = \frac{P_0 V}{R}\left(1 - \frac{V}{V_0}\right) \tag{56}$$

(c) From part (b)

$$\frac{RT}{P_0} = V\left(1 - \frac{V}{V_0}\right)$$

Differentiating with respect to T,

$$\Longrightarrow \frac{R}{P_0} = \frac{dV}{dT}\left(1 - \frac{V}{V_0}\right) + V\left(-\frac{1}{V_0}\frac{dV}{dT}\right)$$

$$= \frac{dV}{dT}\left(1 - \frac{2V}{V_0}\right)$$

Rewriting,

$$\frac{dV}{dT} = \frac{RV_0}{P_0(V_0 - 2V)} \tag{57}$$

(d) From part (c)

$$\frac{dT}{dV} = \frac{P_0}{R}\left(1 - \frac{2V}{V_0}\right)$$

For maximum temperature,

$$\frac{dT}{dV} = 0$$

This occurs at

$$V = V_0/2$$

Hence

$$T_{max} = \frac{P_0 V_0}{4R} \tag{58}$$

(e)

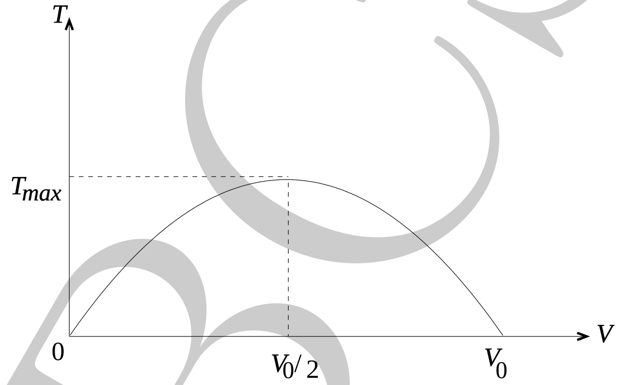


Figure 25: Problem 39 (e)

Eq. (56) is a quadratic in V. See Fig. (25)

(f) Note that

$$C_p - C_v = R$$

and

$$C_v = C_p/\gamma$$

Hence

$$C_v = \frac{R}{\gamma - 1}$$

(g) From the first law of thermodynamics

$$\triangle Q = \triangle U + \triangle W$$

where

$$\triangle W = PdV$$
$$\triangle Q = CdT$$

$$\triangle U = C_v dT$$

Rewriting the first law,

$$CdT = C_v dT + PdV$$

$$\therefore C = C_v + P\frac{dV}{dT}$$

$$= \frac{R}{\gamma - 1} + P_0\left(1 - \frac{V}{V_0}\right)\frac{R}{P_0\left(1 - \frac{2V}{V_0}\right)} \quad \text{(From Eq. (54) and (57))}$$

Rewriting,

$$C = \frac{R}{\gamma - 1} + \frac{(V_0 - V)R}{(V_0 - 2V)} \tag{59}$$

(h) For the mixture of gases the "adiabatic" constant is

$$\gamma = \frac{\sum n_i C_{pi}}{\sum n_i C_{vi}}$$

$$= \frac{\frac{5}{2}R + \frac{7}{2}R}{\frac{3}{2}R + \frac{5}{2}R} \quad \text{(Since } n = 1/2\text{)}$$

$$= \frac{3}{2}$$

(i) Using $\gamma = \frac{3}{2}$ in Eq. (59) we obtain

$$C = R\frac{\left(3 - \frac{5V}{V_0}\right)}{\left(1 - \frac{2V}{V_0}\right)}$$

(j) Note that negative specific heat implies that temperature goes down although heat is being pumped into the system. This is on account of the peculiar linear nature of the process. Also note that temperature too decreases for $V > V_0/2$, the students are advised to calculate.

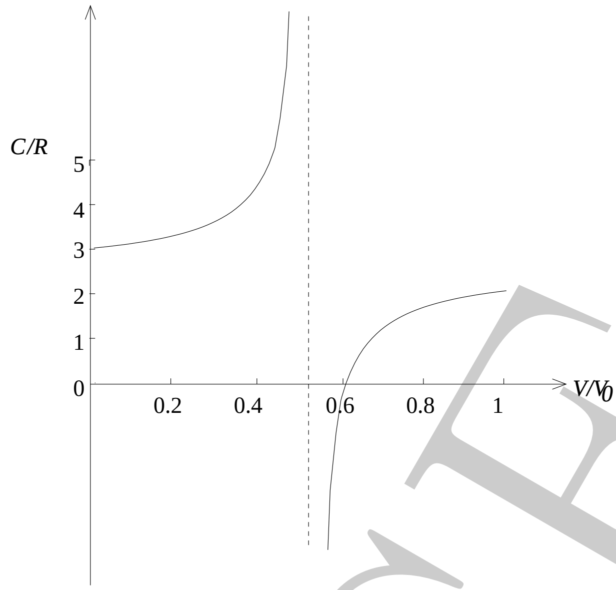


Figure 26: Problem 39 (j)

(29 July 1898-11 Jan.1988)

"Some of the young people I see, who are very good, take physics... as a system you can do things with, can calculate something with, and they miss... the mystery of it : how very different it is from what you can see, and how profound nature is...There is no good translation of the Yiddish word '*Witz*'. It's a joke or a trick or a sleight of hand. You can always bulldoze your way to an answer, but it's the use of this kind of witty trick or subtle approach that I have always liked about physics.... I have always taken physics personally....It's been me and nature and nature is sacred and profound."

-Isidor Isaac Rabi, Nobel Laureate, 1944 on how not to approach a physics problem and how to view nature with respect and awe.

Appendix A

List of Acronyms

HBCSE	Homi Bhabha Centre for Science Education
TIFR	Tata Institute of Fundamental Research
IAPT	Indian Association of Physics Teachers
IACT	Indian Association of Chemistry Teachers
IATBS	Indian Association of Teachers in Biological Sciences
NSE	National Standard Examination
NSEP	National Standard Examination in Physics
NSEC	National Standard Examination in Chemistry
NSEB	National Standard Examination in Biology
INO	Indian National Olympiads
INPhO	Indian National Physics Olympiads
INChO	Indian National Chemistry Olympiads
INBO	Indian National Biology Olympiads
OCSC	Orientation cum Selection Camp
PDT	Pre-departure Training Camp
IPhO	International Physics Olympiad
IChO	International Chemistry Olympiad
IBO	International Biology Olympiad
NCERT	National Council of Education Research and Training
CBSE	Central Board of Secondary Education
IIT-JEE	Indian Institute of Technology - Joint Entrance Examination
AIIMS	All India Institute of Medical Sciences
DAE	Department of Atomic Energy
DST	Department of Science and Technology
MHRD	Ministry of Human Resource Development
BRNS	Board of Research in Nuclear Sciences

Appendix B

IPhO and INPhO Syllabi

International Physics Olympiad Syllabus

General

a. The extensive use of the calculus (differentiation and integration) and the use of complex numbers or solving differential equations should not be required to solve the theoretical and practical problems.
b. Questions may contain concepts and phenomena not contained in the Syllabus but sufficient information must be given in the questions so that candidates without previous knowledge of these topics would not be at a disadvantage.
c. Sophisticated practical equipment likely to be unfamiliar to the candidates should not dominate a problem. If such devices are used then careful instructions must be given to the candidates.
d. The original texts of the problems have to be set in the SI units.

A. Theoretical Part

The first column contains the main entries while the second column contains comments and remarks if necessary.

1. Mechanics

a) Foundation of kinematics of a point mass	Vector description of the position of the point mass, velocity and acceleration as vectors
b) Newton's laws, inertial systems	Problems may be set on changing mass
c) Closed and open systems, momentum and energy, work, power	
d) Conservation of energy, conservation of linear momentum, impulse	
e) Elastic forces, frictional forces, the law of gravitation, potential energy and work in a gravitational field	Hooke's law, coefficient of friction (F/R = const), frictional forces, static and kinetic, choice of zero of potential energy
f) Centripetal acceleration, Kepler's laws	

2. Mechanics of Rigid Bodies

a) Statics, center of mass, torque	Couples, conditions of equilibrium of bodies
b) Motion of rigid bodies, translation, rotation, angular velocity, angular acceleration, conservation of angular momentum	Conservation of angular momentum about fixed axis only
c) External and internal forces, equation of motion of a rigid body around the fixed axis, moment of inertia, kinetic energy of a rotating body	Parallel axes theorem (Steiner's theorem), additivity of the moment of inertia
d) Accelerated reference systems, inertial forces	Knowledge of the Coriolis force formula is not required

3. Hydromechanics No specific questions will be set on this but students would be expected to know the elementary concepts of pressure, buoyancy and the continuity law.

4. Thermodynamics and Molecular Physics

a) Internal energy, work and heat, first and second laws of thermodynamics	Thermal equilibrium, quantities depending on state and quantities depending on process
b) Model of a perfect gas, pressure and molecular kinetic energy, Avogadro's number, equation of state of a perfect gas, absolute temperature	Also molecular approach to such simple phenomena in liquids and solids as boiling, melting etc.
c) Work done by an expanding gas limited to isothermal and adiabatic processes	Proof of the equation of the adiabatic process is not required
d) The Carnot cycle, thermodynamic efficiency, reversible and irreversible processes, entropy (statistical approach), Boltzmann factor	Entropy as a path independent function, entropy changes and reversibility, quasistatic processes

5. Oscillations and waves

a) Harmonic oscillations, equation of harmonic oscillation	Solution of the equation for harmonic motion, attenuation and resonance - qualitatively

b) Harmonic waves, propagation of waves, transverse and longitudinal waves, linear polarization, the classical Doppler effect, sound waves	Displacement in a progressive wave and understanding of graphical representation of the wave, measurements of velocity of sound and light, Doppler effect in one dimension only, propagation of waves in homogeneous and isotropic media, reflection and refraction, Fermat's principle
c) Superposition of harmonic waves, coherent waves, interference, beats, standing waves	Realization that intensity of wave is proportional to the square of its amplitude. Fourier analysis is not required but candidates should have some understanding that complex waves can be made from addition of simple sinusoidal waves of different frequencies. Interference due to thin films and other simple systems (final formulae are not required), superposition of waves from secondary sources (diffraction)

6. Electric Charge and Electric Field

a) Conservation of charge, Coulomb's law	
b) Electric field, potential, Gauss' law	Gauss' law confined to simple symmetric systems like sphere, cylinder, plate etc., electric dipole moment
c) Capacitors, capacitance, dielectric constant, energy density of electric field	

7. Current and Magnetic Field

a) Current, resistance, internal resistance of source, Ohm's law, Kirchhoff's laws, work and power of direct and alternating currents, Joule's law	Simple cases of circuits containing non-ohmic devices with known V-I characteristics
b) Magnetic field (B) of a current, current in a magnetic field, Lorentz force	Particles in a magnetic field, simple applications like cyclotron, magnetic dipole moment
c) Ampere's law	Magnetic field of simple symmetric systems like straight wire, circular loop and long solenoid
d) Law of electromagnetic induction, magnetic flux, Lenz's law, self-induction, inductance, permeability, energy density of magnetic field	
e) Alternating current, resistors, inductors and capacitors in AC-circuits, voltage and current (parallel and series) resonances	Simple AC-circuits, time constants, final formulae for parameters of concrete resonance circuits are not required

8. Electromagnetic waves

a) Oscillatory circuit, frequency of oscillations, generation by feedback and resonance	
b) Wave optics, diffraction from one and two slits, diffraction grating, resolving power of a grating, Bragg reflection,	
c) Dispersion and diffraction spectra, line spectra of gases	
d) Electromagnetic waves as transverse waves, polarization by reflection, polarizers	Superposition of polarized waves
e) Resolving power of imaging systems	
f) Black body, Stefan-Boltzmann law	Planck's formula is not required

9. Quantum Physics

a) Photoelectric effect, energy and impulse of the photon	Einstein's formula is required
b) De Broglie wavelength, Heisenberg's uncertainty principle	

10. Relativity

a) Principle of relativity, addition of velocities, relativistic Doppler effect
b) Relativistic equation of motion, momentum, energy, relation between energy and mass, conservation of energy and momentum

11. Matter

a) Simple applications of the Bragg equation
b) Energy levels of atoms and molecules (qualitatively), emission, absorption, spectrum of hydrogen like atoms
c) Energy levels of nuclei (qualitatively), alpha-, beta- and gamma-decays, absorption of radiation, halflife and exponential decay, components of nuclei, mass defect, nuclear reactions

Indian National Physics Olympiad Syllabus

This is broadly equivalent to senior secondary level (Class XI and Class XII) of the Central Board of Secondary Education (CBSE). For example it does not include Fermat's principle and special relativity. Some of the problems are unconventional, of high difficulty level, and comparable to the International Olympiads.

Appendix C

The Stages of the Indian Physics Olympiad Program

The Olympiad programme is a 5 STAGE process for each subject separately. Stage I for each subject is organized by the Indian Association of Physics Teachers (IAPT) with the assistance of Indian Association Chemistry Teachers (IACT) and Indian Association of Teachers in Biological Sciences (IATBS). All the subsequent stages are organized by the Homi Bhabha Centre for Science Education (HBCSE).

Stage I : National Standard Examinations (NSEs) NSEs are usually conducted in the last week of November at about 1000 centres all over India. Over 35,000 students enroll in the National Standard Examination in Physics (NSEP).

Stage II : Indian National Olympiad Examinations (INOs) Around 300 meritorious students from NSEs are selected for Indian National Olympiad (INO) examination in each subject. These examinations are usually conducted either in the last week of January or in the first week of February at about 15 centres in the country.

Stage III : Orientation Cum Salection Camp (OCSC) About 35 students in each subject are chosen on the basis of their performance in INO exams. The selected group of students in each subject are invited to the OCSC for two to three weeks which are usually held in April-June. Five best students in Physics (four in Chemistry and Biology each) are selected to represent India at respective International Olympiads.

Stage IV : Pre-Departure Training Camp (PDT) The selected Indian teams undergo rigorous training before departing for International Olympiads.

Stage V : Participation in International Olympiads Selected students and 2 to 3 teacher leaders and scientific observers constitute the delegation to represent India at the International Olympiads which are normally held in July.

Information regarding Stage I is available on
IAPT website - **http://www.iapt.org.in**
and information regarding stages II to V and details of eligibility for various stages are available on
HBCSE website - **http://olympiads.hbcse.tifr.res.in**.

Appendix D

Performance of The Indian Team

1998

Dates	Place	Student's Name	Place	Medals
July 2, 1998 to July 10, 1998	Reykjavik, Iceland	Abhishek Kumar	Bokaro	Silver
		Vijay Bhat	Kolkata	Bronze
		Shivi Kumar Bansal	Obra (U.P.)	Honourable Mention
		Dilys Thomas	Pune	Honourable Mention
		Saikat Guha	Patna	Honourable Mention

Delegation Leader : Prof. H.C.Pradhan (HBCSE, Mumbai)
Pedagogical Leader : Prof. T. S. Natrajan (IIT, Chennai)

1999

Dates	Place	Student's Name	Place	Medals
July 18, 1999 to July 27, 1999	Padua, Italy	Sandeep Bala	Mumbai	Silver
		Harsh Madhyastha	Bangalore	Silver
		Mayank Rawat	Panchkula	Silver
		Amit Agarwal	Chandigarh	Silver
		Suvrat Raju	Delhi	Bronze

Delegation Leader : Prof. R. M. Dharkar (IAPT)
Pedagogical Leader : Prof. Vijay A. Singh (IIT, Kanpur)

2000

Dates	Place	Student's Name	Place	Medals
July 8, 2000 to July 16, 2000	Leicester, U.K.	Navneet Loiwal	Jaipur	Gold
		M. Arvind	Chennai	Gold
		Abhineet Sawa	Rourkela	Silver
		Nipun Kwatra	Chandigarh	Silver
		V. Srikant	Hyderabad	Honourable Mention

Delegation Leader : Prof. Vijay A. Singh (IIT, Kanpur)
Pedagogical Leader : Prof. D. A. Desai (D. G. Ruparel College, Mumbai)
Scientific Observer : Prof. Arvind Kumar (HBCSE, Mumbai)

2001

Dates	Place	Student's Name	Place	Medals
June 28, 2001 to July 6, 2001	Antalya, Turkey	Nandan Dixit	Mumbai	Gold
		Arvind Thiagarajan	Chennai	Gold
		Parag Agarwal	Mumbai	Gold
		Naresh Satyan	Bangalore	Silver
		S. Vijaykumar	Bangalore	Silver

Delegation Leader : Prof. Vijay A. Singh (IIT, Kanpur)
Pedagogical Leader : Prof. D. A. Desai (D. G. Ruparel College, Mumbai)

2002

Dates	Place	Student's Name	Place	Medals
July 21, 2002 to July 30, 2002	Bandung, Indonesia	Ravishankar Sundararamam	Mumbai	Gold
		Shantanu Bharadwaj	Mathura	Silver
		Hirakendu Das	Hyderabad	Silver
		B. Sundeep	Bangalore	Silver
		Kushal Mukherjee	Bangalore	Silver

Delegation Leader : Prof. Arvind Kumar (HBCSE, Mumbai)
Pedagogical Leader : Dr. Ravi Bhattacharjee (SGTB Khalsa College, Delhi)

2003

Dates	Place	Student's Name	Place	Medals
August 3, 2003 to August 11, 2003	Taipei, Taiwan	Yashodhan Kanoria	Mumbai	Gold
		Shaleen Harlalka	Udaypur	Gold
		Shashank Dwivedi	Bhilai	Bronze
		Alekh Agarwal	Bhopal	Honourable Mention
		Divjyot Sethi	Delhi	Honourable Mention

Delegation Leader : Dr. Ravi Bhattacharjee (SGTB Khalsa College, Delhi)
Pedagogical Leader : Dr. S. C. Samanta (Midnapore College, Midnapore)

2004

Dates	Place	Student's Name	Place	Medals
July 15, 2004 to July 23, 2004	Pohang, South Korea	Shubham Mittal	New Delhi	Gold
		Ajit Kumar Nema	Bangalore	Silver
		Kartik Mohta	Nagpur	Silver
		Avin Mittal	Agra	Bronze
		Ankur Goel	Panchkula	Bronze

Delegation Leader : Prof. Dipan Ghosh (IIT, Mumbai)
Pedagogical Leader : Dr. Rajesh Khaparde (HBCSE, Mumbai)

2005

Dates	Place	Student's Name	Place	Medals
July 3, 2005 to July 12, 2005	Salamanca, Spain	Piyush Srivastav	Allahabad	Gold
		Sameer Madan	Panchkula	Gold
		Tejaswi Venumadhavan Nerella	Hyderabad	Silver
		Hema Chandra Prakash Movva	Hyderabad	Silver
		Arjun Radhakrishna	Bangalore	Bronze

Delegation Leader : Dr. Ravi Bhattacharjee (SGTB Khalsa College, Delhi)
Pedagogical Leader : Dr. Rajesh Khaparde (HBCSE, Mumbai)
Scientific Observer : Dr. Bhupati Chakravarti (City College, Kolkata)

2006

Dates	Place	Student's Name	Place	Medals
July 8, 2006 to July 17, 2006	Singapore	Mehul Tikekar	Mumbai	Gold
		Raghu Mahajan	Chandigarh	Gold
		Harish Ravi	Bangalore	Bronze
		Divyanshu Jha	Patna	Bronze
		Neha Rambhia	Mumbai	Bronze

Delegation Leader : Dr. Charudatt Kadolkar (IIT, Guwahati)
Pedagogical Leader : Prof. B. N. Chandrika (VVSFG College for Women, Bangalore)
Scientific Observer : Mr. Shirish Pathare (HBCSE, Mumbai)

2007

Dates	Place	Student's Name	Place	Medals
July 13, 2007 to July 22, 2007	Isfahan, Iran	Raman Sharma	Jaipur	Gold
		Rohit Singh	Dehradun	Gold
		Pratyush Pandey	Jaipur	Silver
		Harsh Pareek	Mumbai	Silver
		Vivek Lohani	Almora	Honourable Mention

Delegation Leader : Dr. Charudatt Kadolkar (IIT, Guwahati)
Pedagogical Leader : Prof. Vijay A. Singh (HBCSE, Mumbai)

2008

Dates	Place	Student's Name	Place	Medals
July 20, 2008 to July 29, 2008	Hanoi, Vietnam	Garvit Juniwal	Jaipur	Gold
		Kunal Yogen Shah	Mumbai	Gold
		Nishant Totla	Aurangabad	Gold
		Shitikanth	Patna	Gold
		Saurabh Goyal	Kota	Silver

Delegation Leader : Dr. Pramendra R. Singh (Jagdam College, J. P. University, Chhapra)
Pedagogical Leader : Dr. Vishwajeet Kulkarni (Smt. Parvatibai Chowgule College, Goa)
Scientific Observer : Dr. Charudatt Kadolkar (IIT, Guwahati)

2009

Dates	**Place**	**Student's Name**	**Place**	**Medals**
July 12, 2009 to July 19, 2009	Merida Yucatan, Mexico	Gopi Sivakanth	Yeleswaram	Gold
		Nitin Jain	Faridabad	Gold
		Priyank Pradeep Parikh	Mumbai	Gold
		Shubham Tulsiani	Jodhpur	Gold
		Vinit Atal	Pune	Silver

Delegation Leaders : (1) Prof. H. C. Pradhan (HBCSE, Mumbai)
: (2) Dr. Pramendra R. Singh (Jagdam College, J. P. University, Chhapra)
Scientific Observer : Shri A. M. Shaker (K. J. Somaiya College, Mumbai)

Like the Sports Olympics, the Olympiads are individual events and there is no official ranking of nations by the International Olympiad Committee. Our ranking though unofficial is based on aggregate national scores.

Year	Countries Participated	Rank
1998	56	10
1999	62	10
2000	64	3
2001	65	4
2002	67	7
2003	54	8
2004	71	9
2005	77	8
2006	89	8
2007	70	6
2008*	82	3
2009*	76	3

* 2008 and 2009 Physics Olympiad team performances (4 Golds and 1 Silver) represent best efforts by Indian team in any of the international olympiads (Astronomy, Mathematics, Physics, Chemistry and Biology) India has participated so far.

Indian National Physics Olympiad – 2017

Date: 29th January 2017 **Solutions** Roll Number: 1 7 _ _ - _ _ _ _ - _ _ _ _

Time : **09:00-12:00 (3 hours)** Maximum Marks: **75**

I **permit/do not permit** (*strike out one*) HBCSE to reveal my academic performance and personal details to a third party.

Besides the International Physics Olympiad (IPhO) 2017, do you also want to be considered for the Asian Physics Olympiad (APhO) 2017? For APhO 2017 and its pre-departure training, your presence will be required in Delhi and Russia from April 26 to May 10, 2017. In principle, you can participate in both olympiads. **Yes/No**.

Full Name (BLOCK letters) Ms./Mr.: ______________________________

Extra sheets attached : [ ] Date Centre(e.g.Jaipur) Signature

(Do not write below this line)

==

Instructions

1. This booklet consists of 6 pages (excluding this sheet) and total of 6 questions.
2. This booklet is divided in two parts: **Questions with Summary Answer Sheet** and **Detailed Answer Sheet**. Write roll number at the top wherever asked.
3. **The final answer to each sub-question should be neatly written in the box provided below each sub-question in the Questions & Summary Answer Sheet.**
4. You are also required to show your **detailed work** for each question in a reasonably neat and coherent way in the **Detailed Answer Sheet**. You must write the relevant Question Number on each of these pages.
5. Marks will be awarded on the basis of what you write on both the Summary Answer Sheet and the Detailed Answer Sheet. Simple short answers and plots may be directly entered in the Summary Answer Sheet. Marks may be deducted for absence of detailed work in questions involving loner calculations. Strike out any rough work that you do not want to be evaluated.
6. Adequate space has been provided in the answersheet for you to write/calculate your answers. In case you need extra space to write, you may request for additional blank sheets from the invigilator. Write your roll number on the extra sheets and get them attached to your answersheet and indicate number of extra sheets attached at the top of this page.
7. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.
8. Use blue or black pen to write answers. Pencil may be used for diagrams/graphs/sketches.
9. **This entire booklet must be returned at the end of the examination.**

Question	Marks	Score
1	5	
2	7	
3	11	
4	14	
5	15	
6	23	
Total	75	

Table of Constants

Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Planck's constant	h	6.63×10^{-34} J·s
Universal constant of Gravitation	G	6.67×10^{-11} N·m^2·kg^{-2}
Magnitude of electron charge	e	1.60×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Value of $1/4\pi\epsilon_0$		9.00×10^{9} N·m^2·C^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mole^{-1}

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

1. A massive star of mass M is in uniform circular orbit around a supermassive black hole of mass M_b. Initially, the radius and angular frequency of the orbit are R and ω respectively. According to Einstein's theory of general relativity the space around the two objects is distorted and gravitational waves are radiated. Energy is lost through this radiation and as a result the orbit of the star shrinks gradually. One may assume, however, that the orbit remains circular throughout and Newtonian mechanics holds.

   (a) The power radiated through gravitational wave by this star is given by **[1]**

   $$L_G = Kc^x G^y M^2 R^4 \omega^6$$

   where c is the speed of light, G is the universal gravitational constant, and K is a dimensionless constant. Obtain x and y by dimensional analysis.

   Solution: $x = -5,\ y = 1$

   (b) Obtain the total mechanical energy (E) of the star in terms of M, M_b, and R. **[1]**

   Solution: $E = \dfrac{-GM_bM}{2R}$

   (c) Derive an expression for the rate of decrease in the orbital period (dT/dt) in terms of the masses, period T and constants. **[3]**

   Solution: From Kepler's law

   $$T^2 = \frac{4\pi^2R^3}{GM_b}$$

   Using previous part

   $$E = -\frac{(GM_b2\pi)^{2/3}M}{2T^{2/3}}$$

   Also $dE/dt = L_G$. This yields

   $$\frac{dT}{dt} = -\frac{3KG^{5/3}M_b^{2/3}M(2\pi)^{8/3}}{c^5T^{5/3}}$$

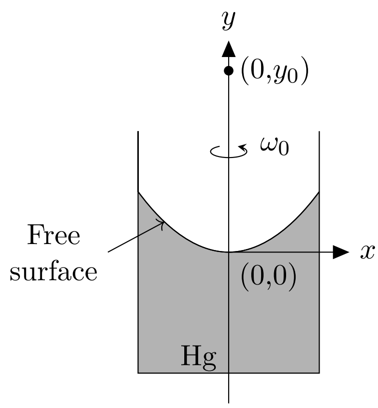
   Answers without −ve sign are also accepted.

Detailed answers can be found on page numbers:________________

2. The free surface of mercury (Hg) is a good reflecting surface. A tall cylinder partly filled with Hg and possessing total moment of inertia I is rotated about its axis with the constant angular velocity ω_0 as shown in figure. The Hg surface attains a paraboloidal profile. The radius of curvature ρ of a general profile is given by

   $$\rho = \left|\frac{[1+(dy/dx)^2]^{3/2}}{d^2y/dx^2}\right|$$

   where the symbols have their usual meaning.



   (a) Obtain the expression for ρ of the Hg surface in terms of ω_0, the distance x from the cylinder axis, and g. **[3]**

Solution:

$$\rho = \frac{\left[1 + \left(\frac{\omega_0^2 x}{g}\right)^2\right]^{3/2}}{\omega_0^2/g}$$

(b) Calculate the value of ρ at the lowest point of the Hg surface, that is (0,0), when $\omega_0 = 78\,\text{rpm}$ (revolutions per minute). **[1]**

Solution:

$$\rho_{x=0} = \frac{g}{\omega_0^2} = 14.7\,\text{cm} \approx 15\,\text{cm}$$

(c) Consider a point object at $(0,y_0)$ as shown in the figure. Obtain an expression for the image position y_i in terms of given quantities. State conditions on y_0 for the formation of real and virtual images. **[3]**

Solution: Using mirror equation

$$\frac{1}{f} = \frac{1}{y_i} - \frac{1}{y_0}$$

where $f = -\rho_{x=0}/2$. This gives

$$y_i = \frac{g y_0}{g - 2\omega_0^2 y_0}$$

For real images $y_0 > g/2\omega_0^2$. For virtual images $y_0 < g/2\omega_0^2$.
A different sign convention can also be used provided it is consistent with the conditions of real and virtual images.

Detailed answers can be found on page numbers:________________

3. Two identical blocks A and B each of mass M are placed on a long inclined plane (angle of inclination $= \theta$) with A higher up than B. The coefficients of friction between the plane and the blocks A and B are respectively μ_A and μ_B with $\tan\theta > \mu_B > \mu_A$. The two blocks are initially held fixed at a distance d apart. At $t = 0$ the two blocks are released from rest.

(a) At what time t_1 will the two blocks collide? **[2]**

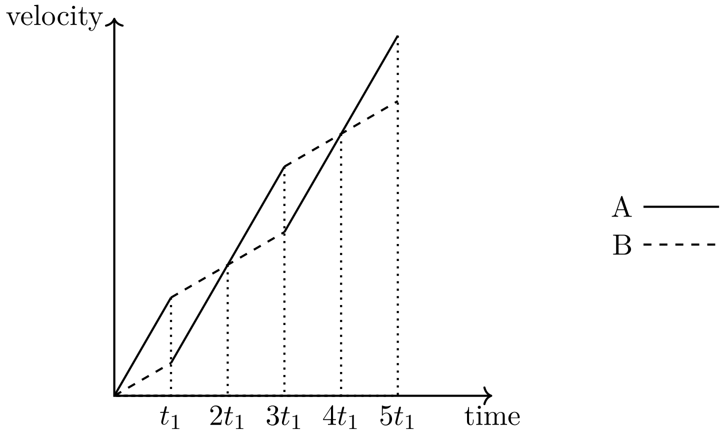
Solution: $t_1 = \sqrt{\dfrac{2d}{(\mu_B - \mu_A) g \cos\theta}}$

(b) Consider each collision to be elastic. At what time t_2 and t_3 will the blocks collide a second and third time respectively? **[4]**

Solution: $t_2 = 3t_1,\ t_3 = 5t_1$

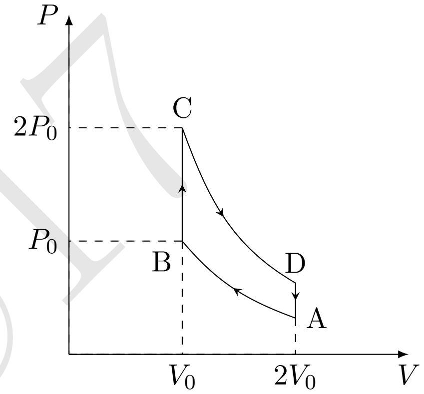
(c) Draw a schematic velocity-time diagram for the two blocks from $t = 0$ till $t = t_3$. Draw below them on a single diagram and use solid line (——) to depict block A and dashed line (- - - -) to depict block B. **[5]**

Solution:



Detailed answers can be found on page numbers:________________

4. One mole of an ideal gas ($c_p/c_v = \gamma$ where symbols have their usual meanings) is subjected to an Otto cycle (A-B-C-D) as shown in the following P-V diagram. Path A-B and C-D are adiabats. The temperature at B is $T_B = T_0$. Diagram is not to scale.



(a) Find the temperatures at A,C, and D in terms of T_0 and pressures at A and D in terms of P_0. **[4]**

Solution: $T_A = \dfrac{T_0}{2^{\gamma-1}}$; $T_C = 2T_0$; $T_D = \dfrac{T_0}{2^{\gamma-2}}$; $P_A = \dfrac{P_0}{2^{\gamma}}$; $P_D = \dfrac{P_0}{2^{\gamma-1}}$

(b) Find total heat absorbed (ΔQ) by the system, the total work done (ΔW) and efficiency (η) of the Otto cycle in terms of γ and related quantities. **[3½]**

Solution:

$$\Delta Q = c_v(T_C - T_B) + c_v(T_A - T_D) = c_v T_0 \left(1 - \frac{1}{2^{\gamma-1}}\right)$$

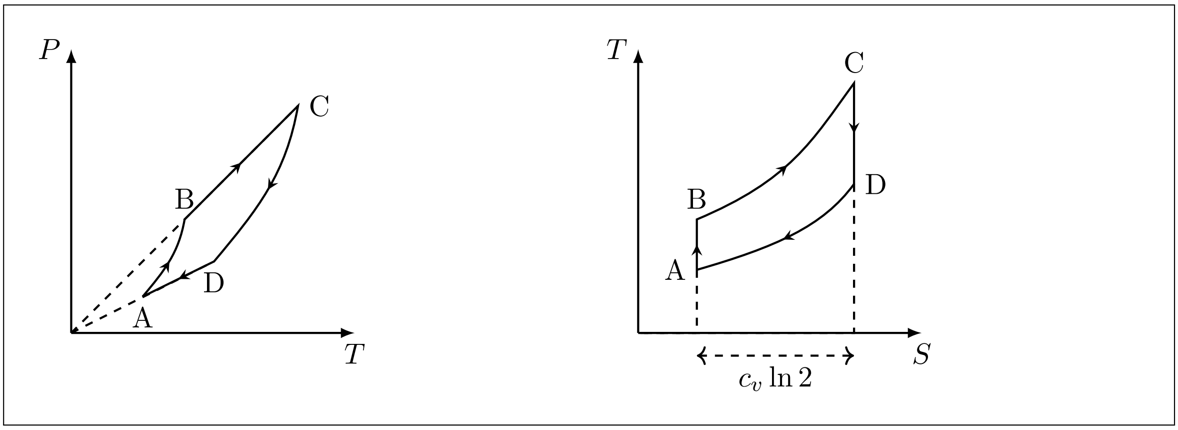
$$\Delta W = \Delta Q$$

$$\eta = 1 - \frac{Q_{BC}}{Q_{AD}} = 1 - \frac{1}{2^{\gamma-1}}$$

$\Delta Q = c_v T_0$ is also accepted provided η is correctly obtained.

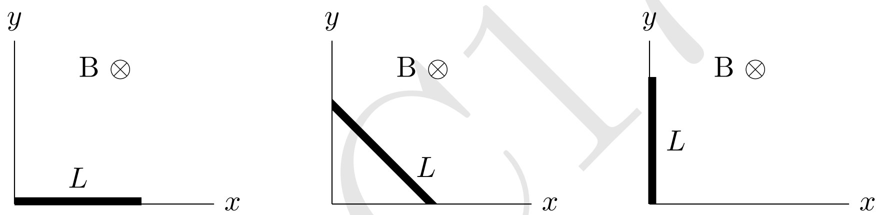
(c) Draw below corresponding P-T and T-S(entropy) diagrams for the cycle. **[6½]**

Solution:



Detailed answers can be found on page numbers:________________

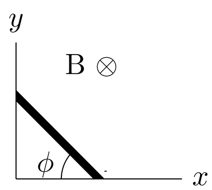
5. A metallic rod of mass m and length L (thick line in the figure below) can slide without friction on two perpendicular wires (thin lines in the figures). Entire arrangement is located in the horizontal plane. A constant magnetic field of magnitude B exists perpendicular to this plane in the downward direction. The wires have negligible resistance compared to the rod whose resistance is R. Initially, the rod is along one of the wires so that one end of it is at the junction of the two wires (see Fig. (a)). **[15]**



(a) (b) (c)

The rod is given an initial angular speed Ω such that it slides with its two ends always in contact with the two wires (see Fig. (b)), and just comes to rest in an aligned position with the other wire (see Fig. (c)). Determine Ω. Neglect the self-inductance of the system.

Solution:



An intermediate position of the rod is shown in figure. The coordinates of the centre of mass of the rod are given by

$$x_{\text{cm}} = \frac{L}{2}\cos\phi \quad (1)$$

$$y_{\text{cm}} = \frac{L}{2}\sin\phi \quad (2)$$

Thus the kinetic energy T of the rod at any instant is

$$T = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\dot{\phi}^2 = \frac{mL^2\omega^2}{6} \quad (3)$$

Magnitude of the induced emf $\mathcal{E}$, is given by

$$\mathcal{E} = \frac{1}{2}BL^2\cos(2\phi)\dot{\phi} \tag{4}$$

The power dissipated due to the current is equal to the loss of kinetic energy of the rod. We have

$$-\frac{dT}{dt} = \frac{\mathcal{E}^2}{R} \tag{5}$$

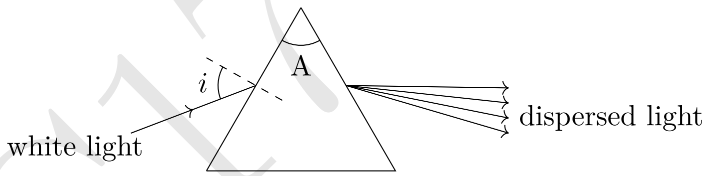
$$\int_{\Omega}^{0}\dot{\phi} = -\frac{3B^2L^2}{4mR}\int_0^{\frac{\pi}{2}}\cos^2(2\phi)d\phi \tag{6}$$

$$\implies \Omega = \frac{3\pi B^2L^2}{16mR} \tag{7}$$

Above solution can also be obtained calculating torque on the rod. Correct methods are accepted.

Detailed answers can be found on page numbers:________________

6. White light is incident at an angle i on a prism of angle A placed in air as shown. Let D be the angular deviation (not necessarily a minimum) suffered by an emergent ray of a particular wavelength.



(a) Obtain an expression for $\sin(D + A - i)$ in terms of the refractive index n and trigonometric functions of i and A only. **[3]**

Solution: $\sin(D + A - i) = n\sin\left(A - \sin^{-1}\frac{\sin i}{n}\right)$

(b) Let $A = 60.00°$ and $i = 45.62°$. Obtain the refractive index (n_λ) for a ray of wavelength λ which has suffered deviation $D = 49.58°$. **[2]**

Solution: $n_\lambda = 1.615$

(c) A detailed microscopic theory yields the relation between the refractive index, n, of the material of the prism and the angular frequency $\omega = 2\pi c/\lambda$ of the incident light as **[10]**

$$\frac{n^2-1}{n^2+2} = \frac{Ne^2}{3\epsilon_0 m_e}\left(\frac{1}{\omega_0^2-\omega^2}\right)$$

Here N is the electron density and $\omega_0 = 2\pi c/\lambda_0$ the natural frequency of oscillation of the electron of the material. The other symbols have their usual meaning. The table below lists the refractive indices at six wavelengths.

λ (nm)	706.54	667.82	501.57	492.19	447.15	438.79
n	1.6087	1.6108	1.6263	1.6277	1.6358	1.6376

Re-express the above equation to get a linear relationship in terms of $\beta = (n^2+2)/(n^2-1)$ and a suitable power of λ. Tabulate and plot so that you may obtain N and ω_0. (Two graph papers are provided with this booklet in case you make a mistake).

Solution: Linear relation:
$$\frac{1}{\lambda^2} = \frac{1}{\lambda_0^2} - \beta\frac{Ne^2}{3\epsilon_0 m_e(2\pi c)^2}.$$
Any linear form of above equation is accepted.
Graph is plotted between β vs $1/\lambda^2$ at the end of this booklet.

$1/\lambda^2$ ($\times 10^{-6}\,\mathrm{nm}^{-2}$)	2.0032	2.2422	3.9750	4.1280	5.0014	5.1938
β	2.8893	2.8813	2.8239	2.8188	2.7901	2.7839

(d) Calculate the values of N, ω_0 from the graph you plotted. Which part of the electromagnetic spectrum does λ_0 belong to? **[4]**

Solution: From the drawn graph, $N = 1.01 \times 10^{29}\mathrm{m}^{-3}$; $\omega_0 = 1.78 \times 10^{16}$ Hz. λ_0 belongs to the ultraviolet part of the electromagnetic spectrum. Accepted values:
$9.90 \times 10^{28} \leq N \leq 1.10 \times 10^{29}\,\mathrm{m}^{-3}$ and $1.68 \times 10^{16} \leq \omega_0 \leq 1.88 \times 10^{16}$ Hz.

(e) An X-ray of energy 1.000 keV is incident on the prism. If we write $n = 1 + \delta$ then obtain the numerical value of δ for this ray. **[3]**

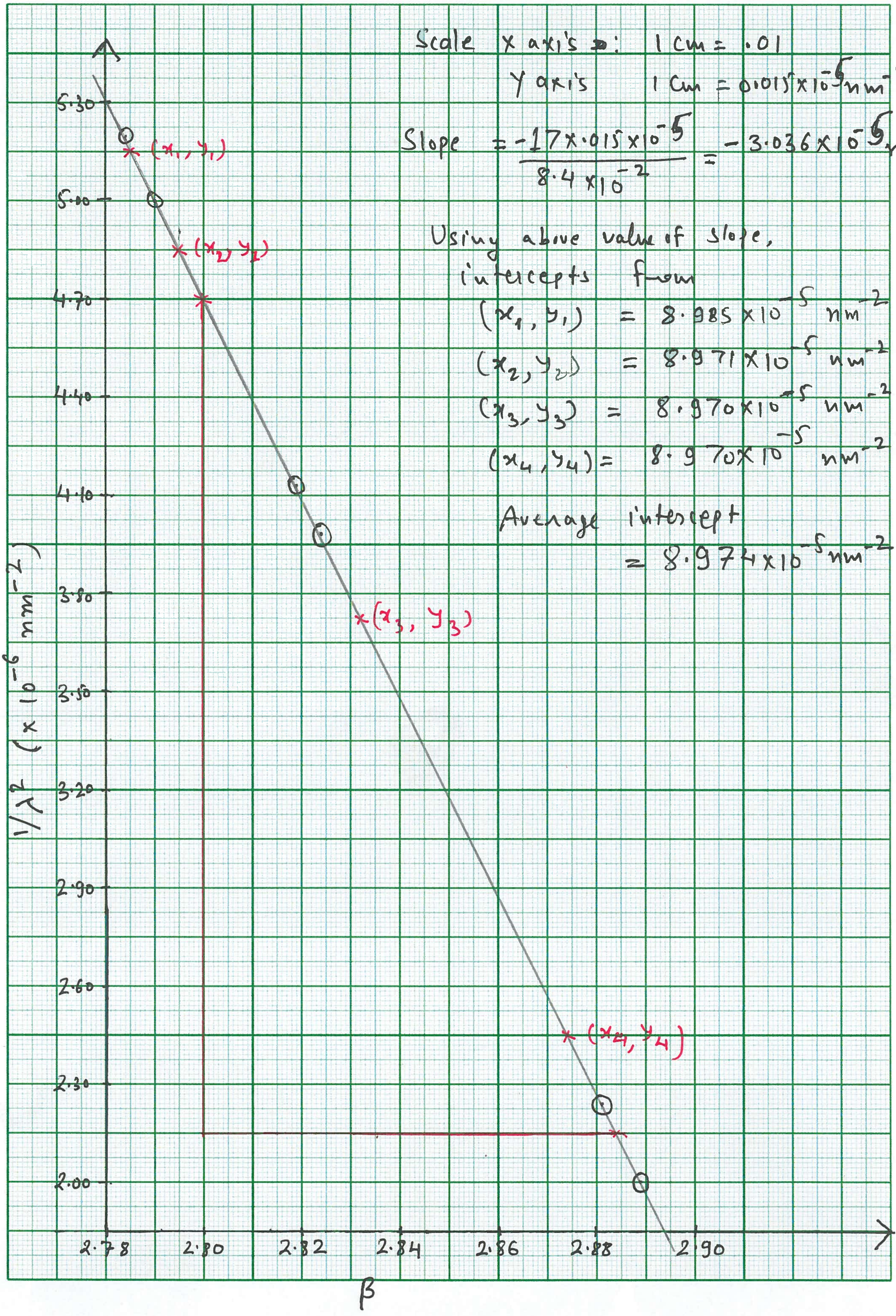
Solution: $\delta \approx -0.70 \times 10^{-4}$

(f) For the X-ray of the previous part let i_c be the critical angle and $\theta_c = 90 - i_c$ be the corresponding grazing angle. Obtain θ_c. **[1]**

Solution: $\theta_c = 0.68°$

Detailed answers can be found on page numbers:________________

****** END OF THE QUESTION PAPER ******



Scale x axis: 1 cm = .01

y axis 1 cm = $0.015 \times 10^{-5}\ nm^{-2}$

$$\text{Slope} = \frac{-17 \times .015 \times 10^{-5}}{8.4 \times 10^{-2}} = -3.036 \times 10^{-5}\ nm^{-2}$$

Using above value of slope, intercepts from

$(x_1, y_1) = 8.985 \times 10^{-5}\ nm^{-2}$

$(x_2, y_2) = 8.971 \times 10^{-5}\ nm^{-2}$

$(x_3, y_3) = 8.970 \times 10^{-5}\ nm^{-2}$

$(x_4, y_4) = 8.970 \times 10^{-5}\ nm^{-2}$

Average intercept

$= 8.974 \times 10^{-5}\ nm^{-2}$

Solutions of Indian National Physics Olympiad – 2019

Date: 03 February 2019 | Roll Number: 1 9 _ _ - _ _ _ _ - _ _ _ _

Time : **09:00-12:00 (3 hours)** | Maximum Marks: **75**

Extra sheets attached : ☐ | ______ Centre (e.g. Kota)

==================*(Do not write below this line)*==================

Instructions

1. This booklet consists of 20 pages (excluding this page) and total of 7 questions.
2. This booklet is divided in two parts: **Questions with Summary Answer Sheet** and **Detailed Answer Sheet**. Write roll number at the top wherever asked.
3. **The final answer to each sub-question should be neatly written in the box provided below each sub-question in the Questions & Summary Answer Sheet.**
4. You are also required to show your **detailed work** for each question in a reasonably neat and coherent way in the **Detailed Answer Sheet**. You must write the relevant Question Number(s) on each of these pages.
5. Marks will be awarded on the basis of what you write on both the Summary Answer Sheet and the Detailed Answer Sheet. Simple short answers and plots may be directly entered in the Summary Answer Sheet. Marks may be deducted for absence of detailed work in questions involving longer calculations. Strike out any rough work that you do not want to be considered for evaluation.
6. Adequate space has been provided in the answersheet for you to write/calculate your answers. In case you need extra space to write, you may request additional blank sheets from the invigilator. Write your roll number on the extra sheets and get them attached to your answersheet and indicate number of extra sheets attached at the top of this page.
7. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.
8. Use blue or black pen to write answers. Pencil may be used for diagrams/graphs/sketches.
9. **This entire booklet must be returned at the end of the examination.**

Table of Constants

Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Planck's constant	h	6.63×10^{-34} J·s
	$\hbar$	$h/2\pi$
Universal constant of Gravitation	G	6.67×10^{-11} N·m^2·kg^{-2}
Magnitude of electron charge	e	1.60×10^{-19} C
Rest mass of electron	m_e	9.11×10^{-31} kg
Value of $1/4\pi\epsilon_0$		9.00×10^{9} N·m^2·C^{-2}
Avogadro's number	N_A	6.022×10^{23} mol^{-1}
Acceleration due to gravity	g	9.81 m·s^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mol^{-1}
	R	0.0821 l·atm·mol^{-1}·K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H·m^{-1}

Question	Marks	Score
1	9	
2	11	
3	12	
4	7	
5	9	
6	14	
7	13	
Total	75	

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

1. In the lower part of the earth's atmosphere, the temperature decreases with increase of height. Choose the origin of the coordinate system at the ground level with the y- axis vertically upward and the x-axis horizontal. We assume a linear decrease of temperature such that the temperature at a height y from the ground level is

$$T(y) = T_0(1 - by)$$

where T_0 is the temperature at the ground level. The constant $b = 0.023\,\mathrm{km}^{-1}$. We consider the propagation of sound in the x-y plane. Ignore any attenuation, reflection, and diffraction of sound.

(a) If v_0 is the speed of sound at the ground level, obtain an expression for the speed of sound $v(y)$ at height y, in terms of v_0 and b. **[1]**

$v(y)$=

Solution: Speed of sound is

$$v = \sqrt{\frac{\gamma RT}{m}} \tag{1.1}$$

where m and γ are the molar mass and adiabatic index of the gas respectively.

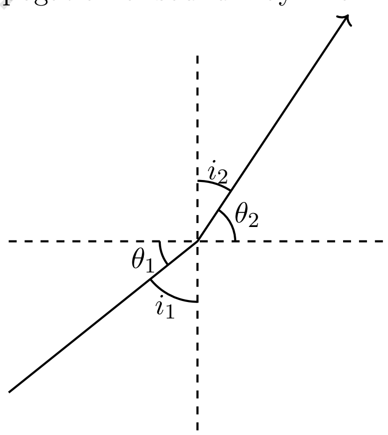
$$v(y) = \sqrt{\frac{\gamma RT(y)}{m}} = \sqrt{\frac{\gamma RT_0}{m}}\sqrt{1 - by} = v_0\sqrt{1 - by} \tag{1.2}$$

Here $v_0 = \sqrt{\gamma RT_0/m}$ is the speed of the sound at ground level.

(b) Suppose sound propagates from the origin with an initial angle θ_0 with the x-axis. Obtain an expression for the angle θ made by the direction of propagation of sound with the horizontal at height y, in terms of θ_0 and b. **[2]**

θ =

Solution: Consider the propogation of sound "ray" from one medium to the other.



From Snell's law,

$$\frac{\sin i_2}{\sin i_1} = \frac{v_2}{v_1} \tag{1.3}$$

$$\frac{\sin(\frac{\pi}{2} - \theta_2)}{\sin(\frac{\pi}{2} - \theta_1)} = \frac{\cos\theta_2}{\cos\theta_1} = \frac{v_2}{v_1} \tag{1.4}$$

$$\frac{\cos\theta}{\cos\theta_0} = \frac{v(y)}{v_0} = \sqrt{1-by} \tag{1.5}$$

$$\theta = \cos^{-1}\left[\cos\theta_0\sqrt{1-by}\right] \tag{1.6}$$

(c) Obtain an expression for the x and y coordinates of a point on the path of propagation as functions of θ. **[3]**

$x =$

$y =$

Solution:

$$y = \frac{1}{b}\left(1 - \frac{\cos^2\theta}{\cos^2\theta_0}\right) \tag{1.7}$$

$$\frac{dy}{dx} = \tan\theta \tag{1.8}$$

$$dx = \frac{2\cos\theta\sin\theta}{\tan\theta\, b\cos^2\theta_0}d\theta \tag{1.9}$$

$$x = \int_0^x dx = \frac{1}{2b\cos^2\theta_0}\left[2(\theta-\theta_0) + \sin 2\theta - \sin 2\theta_0\right] \tag{1.10}$$

(d) For this part assume the direction of propagation of sound to be horizontal at the origin. For the case where y is of the order 100 m or less, obtain an approximated expression relating x and y i.e. $y(x)$. Obtain x for $y = 2.00\,\text{m}$. **[3]**

$y(x) =$ Value of $x =$

Solution: For $y \leq 100\,\text{m}$ and $\theta_0 = 0$, Eq. (1.6) gives $\theta \approx 3^\circ$ (very small).

$$y \approx \frac{1}{b}\left[1 - \left(1 - \frac{\theta^2}{2}\right)^2\right] = \frac{1}{b}\theta^2 \tag{1.11}$$

$$x \approx \frac{1}{2b}(2\theta + 2\theta) = \frac{2\theta}{b} \tag{1.12}$$

$$x^2 = \frac{4y}{b} \tag{1.13}$$

$$x(y=2) \approx 590\,\text{m} \tag{1.14}$$

Accepted range 585-600 m.

Detailed answers can be found on page numbers:

2. Consider a particle of mass m confined to a one dimensional box of length L. The particle moves in the box with momentum p colliding elastically with the walls. We consider the quantum mechanics of this system. As far as possible, express your answers in terms of $\alpha = h^2/8m$.

(a) At each energy state, the particle may be represented by a standing wave given by the de Broglie hypothesis. Express its wavelengths λ_{dB} in terms of L in the n^{th} energy state. **[1]**

$\lambda_{\text{dB}} =$

Solution: $\lambda_{\text{dB}} = \dfrac{2L}{n}$

(b) Write the energy of the n^{th} energy state, E_n. **[1]**

$E_n =$

Solution:

$$E_n = \frac{p^2}{2m} = \frac{n^2h^2}{8mL^2} = \frac{\alpha n^2}{L^2} \tag{2.1}$$

where we have used $p = h/\lambda_{\text{dB}}$.

(c) Let there be N (mass m) electrons in this box where N is an even number. Obtain the expression for the lowest possible total energy U_0 of the system (e.g., the ground state energy of this N-particle system). Neglect coulombic interaction between the electrons. **[2]**

$U_0 =$

Solution: Highest occupied level is $n_{\max} = N/2$ and each level is occupied by the two electrons.

$$U_0 = \sum_{n=1}^{N/2} E_n = \frac{2\alpha}{L^2}\sum_{n=1}^{N/2} n^2 = \frac{\alpha N(N+1)(N+2)}{12L^2} \tag{2.2}$$

(d) Express the total energy U_1 in terms of U_0 and relevant quantities when the system is in the first excited state. Also express the total energy U_2 in terms of U_0 and relevant quantities when the system is in the second excited state. **[3½]**

$U_1 =$

$U_2 =$

Solution: 1st excited state configuration is

Energy level	Configuration
$N/2+1$	↿
$N/2$	↿

$$U_1 = U_0 - \frac{\alpha}{L^2}\left(\frac{N}{2}\right)^2 + \frac{\alpha}{L^2}\left(\frac{N}{2}+1\right)^2 \tag{2.3}$$
$$= U_0 + \frac{\alpha}{L^2}(N+1) \tag{2.4}$$

Solution: 2nd excited state configuration is

Energy level	Configuration
$N/2+1$	↿
$N/2$	↿⇂
$N/2-1$	↿

$$U_2 = U_0 - \frac{\alpha}{L^2}\left(\frac{N}{2}-1\right)^2 + \frac{\alpha}{L^2}\left(\frac{N}{2}+1\right)^2 \tag{2.5}$$
$$= U_0 + \frac{\alpha}{L^2}2N \tag{2.6}$$

(e) When the system is in the ground state, let the length of the box change slowly from L to $L-\Delta L$. Obtain the magnitude of the force F on each wall in terms of U_0, when $\Delta L \ll L$. **[1]**

$F =$

Solution: From the work-energy theorem,

$$F\Delta L = U_{\text{final}} - U_{\text{initial}} \tag{2.7}$$
$$= \frac{\alpha N(N+1)(N+2)}{12}\left[\frac{1}{(L-\Delta L)^2} - \frac{1}{L^2}\right] \tag{2.8}$$
$$F \approx \frac{\alpha N(N+1)(N+2)2}{12L^3} = \frac{2U_0}{L} \tag{2.9}$$

(f) Assuming N is large ($N \gg 1$) obtain the ratio r of dU_0/dN to the energy level of the highest occupied ground state. **[1]**

$r =$

Solution:

$$\frac{dU_0}{dN} = \frac{\alpha}{12L^2}\frac{d}{dN}(N^3 + 3N^2 + 2N) \quad (2.10)$$

$$\approx \frac{\alpha}{12L^2}3N^2 \quad (2.11)$$

Highest occupied ground state corresponds to $N/2$, for which the energy level is $\alpha N^2/4L^2$. Thus

$$r = 1 \quad (2.12)$$

(g) We assume once again that N is large. Consider the possibility of the electrons forming a uniform continuum of length L with constant linear density. Using dimensional analysis, calculate the gravitational energy of this system U_G assuming that it depends on its total mass, universal gravitational constant G and L. Equate this (attractive) energy to the (repulsive) energy $U_0(N)$. Obtain L in terms of N and related quantities. **[1½]**

$U_G =$

$L =$

Solution: $[U_G] = \dfrac{GM^2}{L} = \dfrac{GN^2m^2}{L}$ or $-\dfrac{GN^2m^2}{L}$

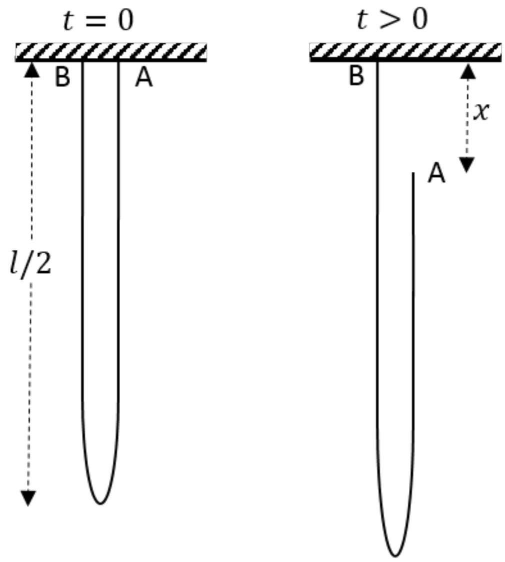
Solution: For large N, $U_0 \approx \alpha N^3/12L^2$.

$$\frac{GN^2m^2}{L} = \frac{\alpha N^3}{12L^2} \quad (2.13)$$

$$L = \frac{\alpha N^3}{12GM^2} = \frac{Nh^2}{96Gm^3} \quad (2.14)$$

Detailed answers can be found on page numbers:

3. A chain of length l and linear density λ hangs from a horizontal support with both ends A and B fixed to a horizontal support as shown. The two fixed ends are close to each other. At time t = 0 the end A is released. All vertical distances (x) are measured with respect to the horizontal support with the downward direction taken as positive (A and B are initially at $x = 0$).



(a) Obtain the momentum P of the center of mass of the system when the end A has fallen by a distance x, in terms of x and speed $\dot{x}$. **[2]**

$P =$

Solution: Let total mass of the chain to be $M = \lambda l$.
The mass of the left side of the chain $= \lambda\left(\frac{l+x}{2}\right)$
CM of the left side of the chain $= \left(\frac{l+x}{4}\right)$
The mass of the right side of the chain $= \lambda\left(\frac{l-x}{2}\right)$
CM of the right side of the chain $= \left(x + \frac{l-x}{4}\right)$

$$x_{\text{CM}} = \frac{\lambda\left(\frac{l+x}{2}\right)\left(\frac{l+x}{4}\right) + \lambda\left(\frac{l-x}{2}\right)\left(x+\frac{l-x}{4}\right)}{\lambda l} \tag{3.1}$$

$$\lambda l x_{\text{CM}} = \frac{\lambda l^2}{4} + \frac{\lambda l x}{2} - \frac{\lambda x^2}{4} \tag{3.2}$$

$$P = \lambda l \dot{x}_{\text{CM}} = \lambda\left(\frac{l-x}{2}\right)\dot{x} \tag{3.3}$$

(b) Assume that the end A is falling freely under gravity, i.e., $\ddot{x} = g$. Obtain the tension T at the fixed end B just before the chain completes the fall and becomes entirely vertical. **[1]**

$T =$

Solution:

$$\ddot{x} = g \tag{3.4}$$

$$\dot{x} = gt = \sqrt{2gx} \tag{3.5}$$

$$\dot{P} = Mg - T \tag{3.6}$$

From Eq. (3.3)

$$\dot{P} = \frac{\lambda}{2}[\ddot{x}(l-x) - \dot{x}^2] \tag{3.7}$$

$$\frac{\lambda}{2}[gl - 3gx] = Mg - T \tag{3.8}$$

$$T(x) = \frac{Mg}{2}\left(1 + \frac{3x}{l}\right) = \frac{\lambda lg}{2}\left(1 + \frac{3x}{l}\right) \tag{3.9}$$

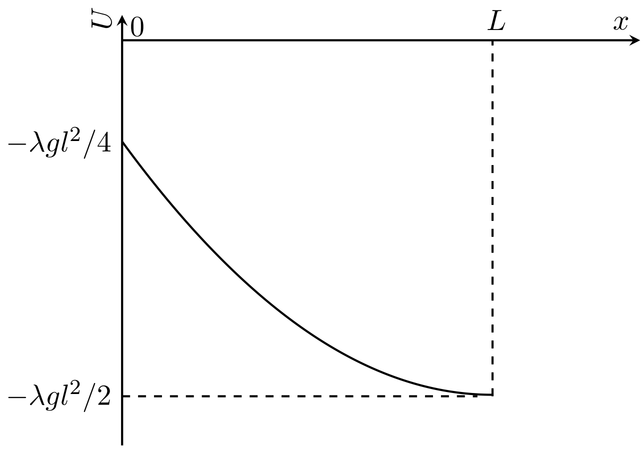
$$T(x = l) = 2\lambda lg \tag{3.10}$$

Experimentally, the value of tension is found to be different from the above result. We adopt an alternative approach assuming conservation of mechanical energy.

(c) Obtain the potential energy $U(x)$ of the chain and plot it versus x. Take the potential energy of a point mass placed at the horizontal support to be zero. **[2½]**

Solution: In general, $U = -\lambda lg x_{\mathrm{CM}}$. Using Eq. (3.2)

$$U = \frac{-\lambda g}{4}(l^2 + 2lx - x^2) \tag{3.11}$$



(d) Obtain the speed $\dot{x}$ when the end A has fallen by a distance x. Assume that all sections of the falling (right side) part of the chain have the same speed. **[2½]**

$\dot{x} =$

Solution: From Eq. (3.3), kinetic energy of the chain

$$K(x) = \frac{1}{2}\lambda\left(\frac{l-x}{2}\right)\dot{x}^2 \tag{3.12}$$

As the total energy is conserved,

$$U(x) + K(x) = U(t=0) \tag{3.13}$$

$$\dot{x}^2 = \left[\frac{g(2lx - x^2)}{(l-x)}\right]^{1/2} \tag{3.14}$$

$$\dot{x} = \left[\frac{g(2lx - x^2)}{(l-x)}\right]^{1/2} \tag{3.15}$$

(e) Hence obtain $T(x)$ at B as a function of x and related quantities. You are advised to simplify your expression as far as possible. **[3]**

$T(x) =$

Solution:

$$\frac{d}{dt}\dot{x}^2 = 2\dot{x}\ddot{x}$$

Substituing Eq. (3.15) in the above equation yields

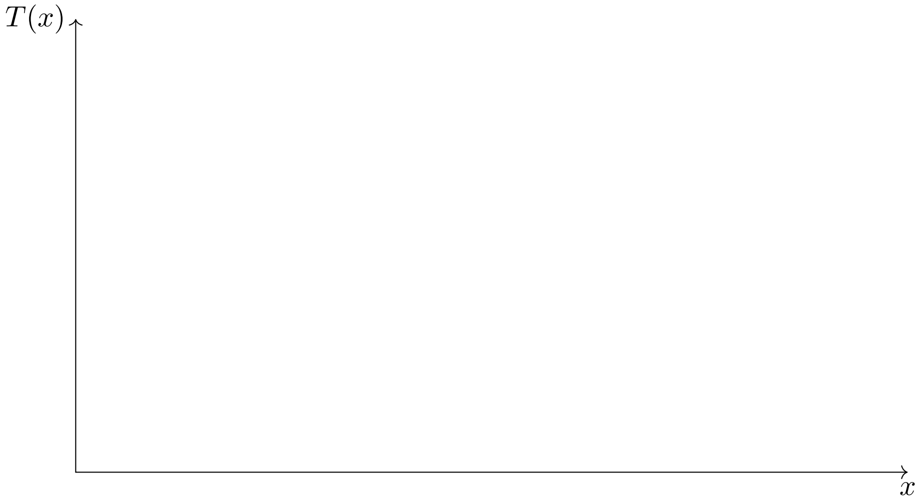
$$\ddot{x} = g + \frac{g(2lx - x^2)}{2(l-x)^2} \tag{3.16}$$

Combining Eqs. (3.6), (3.7) and (3.16)

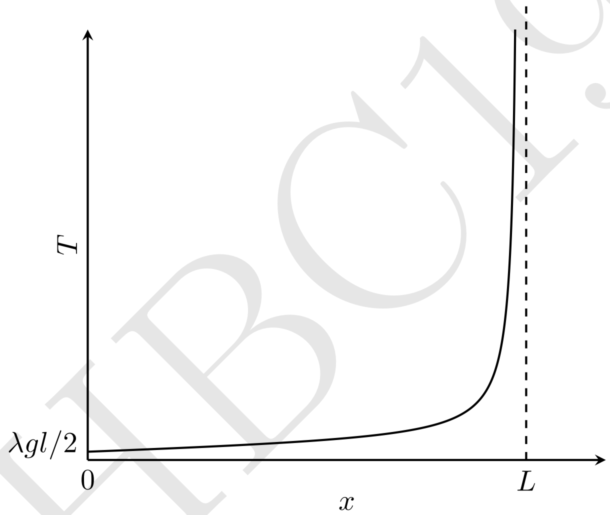
$$T = \lambda lg - \frac{\lambda}{2}[\ddot{x}(l-x) - \dot{x}^2] \tag{3.17}$$

$$= \frac{\lambda g}{4(l-x)}[2l^2 + 2lx - 3x^2] \tag{3.18}$$

(f) Qualitatively sketch $T(x)$ versus x. **[1]**

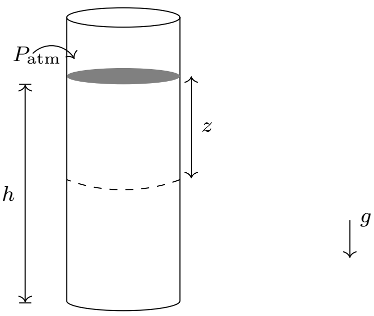


Solution:



Detailed answers can be found on page numbers:

4. Consider a long narrow cylinder of cross section A filled with a compressible liquid up to height h whose density ρ is a function of the pressure $P(z)$ as $\rho(z) = \dfrac{\rho_0}{2}\left(1 + \frac{P(z)}{P_0}\right)$ where P_0 and ρ_0 are constants. The depth z is measured from the free surface of the liquid where the pressure is equal to the atmospheric pressure (P_{atm}).



(a) Obtain the pressure ($P(z)$) as a function of z. Obtain the mass (M) of liquid in the tube. **[5]**

$P(z) =$

Solution:

$$P(z) = \int \rho(z) g\, dz + P_{\text{atm}} \tag{4.1}$$
$$= \int_0^z \frac{\rho_0}{2}\left(1 + \frac{P(z)}{P_0}\right) g\, dz + P_{\text{atm}} \tag{4.2}$$
$$\frac{dP(z)}{dz} = \frac{\rho_0}{2}\left(1 + \frac{P(z)}{P_0}\right) g \tag{4.3}$$
$$\int_{P_{\text{atm}}}^{P(z)} \frac{dP(z)}{P_0 + P(z)} = \int_0^z \frac{\rho_0 g\, dz}{2P_0} \tag{4.4}$$
$$\log \frac{P_0 + P(z)}{P_0 + P_{\text{atm}}} = \frac{\rho_0 g z}{2P_0} \tag{4.5}$$
$$P(z) = \left[(P_0 + P_{\text{atm}}) e^{\rho_0 g z/2P_0} - P_0\right] \tag{4.6}$$

$M =$

Solution:

$$\rho(z) = \frac{\rho_0 (P_0 + P_{\text{atm}})}{2P_0} e^{\rho_0 g z/2P_0} \tag{4.7}$$
$$M = \int_0^h \rho(z) A\, dz \tag{4.8}$$
$$= \frac{A}{g}(P_0 + P_{\text{atm}})(e^{\rho_0 g h/2P_0} - 1) \tag{4.9}$$

(b) Let $P_{\text{i}}(z)$ be the pressure at z, if the liquid were incompressible with density $\rho_0/2$. Assuming that $P_0 \gg \rho_0 g z$ obtain an approximated expression for $\Delta P = P(z) - P_{\text{i}}(z)$. **[2]**

$\Delta P \approx$

Solution:

$$P_i(z) = P_{\text{atm}} + \frac{\rho_0}{2} g z \tag{4.10}$$
$$P(z) = (P_0 + P_{\text{atm}}) e^{\rho_0 g z/2P_0} - P_0 \tag{4.11}$$
$$\approx P_0 \left(1 + \frac{P_{\text{atm}}}{P_0}\right)\left[1 + \frac{\rho_0 g z}{2P_0} + \left(\frac{\rho_0 g z}{2P_0}\right)^2 \frac{1}{2}\right] - P_0 \tag{4.12}$$
$$\Delta P = P(z) - P_i(z) \tag{4.13}$$
$$= \frac{(\rho_0 g z)^2}{8P_0} + \frac{P_{\text{atm}}}{P_0}\frac{\rho_0 g z}{2} + \frac{P_{\text{atm}}}{2}\left(\frac{\rho_0 g z}{2P_0}\right)^2 \tag{4.14}$$

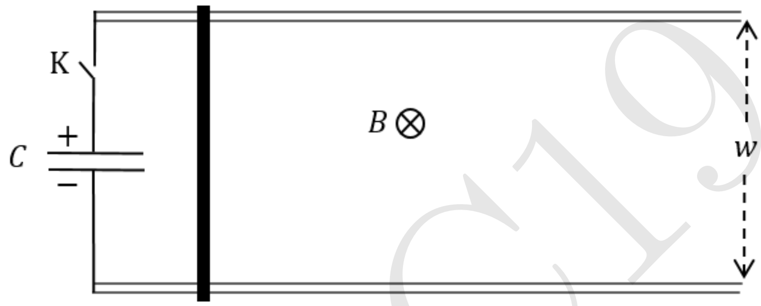
which is correct upto second order in $\rho_0 g z/2P_0$.

$$\Delta P \approx \frac{P_{\mathrm{atm}}}{P_0}\frac{\rho_0 g z}{2} \tag{4.15}$$

which is correct upto first order in $\rho_0 gz/2P_0$.
Both Eqs. (4.14) and (4.15) (mentioning order of approximation) gain full credit.

Detailed answers can be found on page numbers:

5. A pair of long parallel metallic rails of negligible resistance and separation w are placed horizontally. A horizontal metal rod (dark thick line) of mass M and resistance R is placed perpendicularly onto the rails at one end (as shown). A uniform magnetic field B exists perpendicular to the plane of the paper (pointing into the page). One end of the rail track is connected to a key (K) and a capacitor of capacitance C charged to voltage V_0. At $t = 0$ the key is closed. Neglect friction and self inductance of the loop.



(a) What is the final speed (v_{final}) attained by the rod? **[6]**

$v_{\mathrm{final}} =$

Solution: At $t = 0, Q_0 = CV_0$. At any instant, $\dot{Q} = -I$, where Q is the charge on capacitor and I is the current in RC circuit. From Newton's second law and Lorentz force,

$$m\frac{dv}{dt} = IwB \tag{5.1}$$

$$= -\dot{Q}wB \tag{5.2}$$

Induced emf in the circuit is Bwv. From KVL,

$$\frac{Q}{C} = IR + Bwv \tag{5.3}$$

$$= -\dot{Q}R + Bwv \tag{5.4}$$

Differentiating Eq. (5.4) and using Eq. (5.2)

$$\ddot{Q} = -\frac{1}{RC}\dot{Q} - \frac{B^2w^2}{mR}\dot{Q} \tag{5.5}$$

$$= -\dot{Q}\left[\frac{1}{RC} + \frac{B^2w^2}{mR}\right] \tag{5.6}$$

$$\ddot{Q} = -\frac{\dot{Q}}{\tau} \text{ where } \frac{1}{\tau} = \frac{1}{RC} + \frac{B^2w^2}{mR} \tag{5.7}$$

$$\log\left(\frac{\dot{Q}}{\dot{Q}_0}\right) = -\frac{t}{\tau} \text{ where } \dot{Q}_0 = I(t=0) = -\frac{V_0}{R} \tag{5.8}$$

$$\dot{Q} = -\frac{V_0}{R}e^{-t/\tau} \tag{5.9}$$

Combining Eqs. (5.2) and (5.9)

$$m\frac{dv}{dt} = \frac{V_oBw}{R}e^{-t/\tau} \tag{5.10}$$

$$\int_0^{v_{\text{final}}} m\,dv = \int_0^{\infty}\frac{V_0Bw}{R}e^{-t/\tau}dt \tag{5.11}$$

$$mv_{\text{final}} = \frac{BwV_0}{R}(-\tau)e^{-t/\tau}\Big|_0^{\infty} \tag{5.12}$$

$$= \frac{BwV_0\tau}{R} \tag{5.13}$$

$$v_{\text{final}} = \frac{BwCV_0}{m + B^2w^2c} \tag{5.14}$$

(b) Consider the ratio r of the maximum kinetic energy attained by the rod to the energy initially stored in the capacitor. What is r_{max}, the maximum possible value of r, by appropriately choosing B? **[2]**

$r_{\text{max}} =$

Solution:

$$r = \frac{mv_{\text{max}}^2/2}{CV_0^2/2} \tag{5.15}$$

$$= \frac{x}{(1+x)^2} \text{ where } x = \frac{B^2w^2C}{m} \tag{5.16}$$

r has a maximum value of $\frac{1}{4}$ at $x = 1$

$$r_{\text{max}} = 1/4 \tag{5.17}$$

(c) Let $M = 10.0$ kg, $w = 0.10\,\text{m}$, $V_0 = 1.00 \times 10^4\,\text{V}$ and a bank of capacitors ensures that $C = 1.00\,\text{F}$. If $r = r_{\text{max}}$, calculate the value of v_{final}. **[1]**

$v_{\text{final}}(r = r_{\text{max}}) =$

Solution: For $r_{\max}$,

$$\frac{B^2 w^2 C}{m} = 1 \Rightarrow B = 31.6\,\mathrm{T} \tag{5.18}$$

$$v_{\max} = 1.58 \times 10^3\,\mathrm{m/s} \tag{5.19}$$

Detailed answers can be found on page numbers:

6. Consider n moles of a monoatomic non-ideal (realistic) gas. Its equation of state may be described by the van der Waal's equation

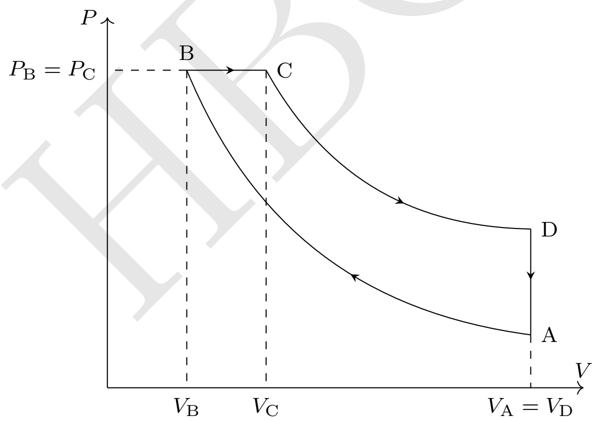
$$\left(P + \frac{an^2}{V^2}\right)\left(\frac{V}{n} - b\right) = RT$$

where a and b are positive constants and other symbols have their usual meanings. The internal energy change of a realistic gas can be given by

$$dU = C_V\,dT + \left\{T\left(\frac{dP}{dT}\right)_V - P\right\}dV$$

As indicated in the above expression, the derivative of pressure is taken at constant volume.

We take one mole of the gas ($n = 1$) through a Diesel cycle (ABCDA) as shown in the following P-V diagram (diagram is not to scale). During the whole cycle assume that the molar heat capacity at constant volume (C_V) remains constant at $3R/2$. Path AB and CD are reversible adiabats.



(a) Obtain the temperature at B (T_B) in terms of temperature at A (T_A), V_A, V_B and constants only. **[2½]**

$T_B =$

Solution:

$$dU = C_V\,dT + \left\{\frac{RT}{V-b} - P\right\}dV$$

AB and CD are reversible adiabats, hence entropy change during these processes are zero.

$$\Delta S_{\text{AB}} = \Delta S_{\text{CD}} = \int_A^B \frac{dQ}{T} = \int_C^D \frac{dQ}{T} = 0 \tag{6.1}$$

$$\int_{\text{A}}^{\text{B}} \frac{dQ}{T} = \int \frac{C_V}{T} dT + \int \frac{R}{V-b} dV = 0 \tag{6.2}$$

$$C_V \ln \frac{T_{\text{B}}}{T_{\text{A}}} = -R \ln \left(\frac{V_{\text{B}} - b}{V_{\text{A}} - b} \right) \tag{6.3}$$

$$T_{\text{B}} = T_{\text{A}} \left(\frac{V_{\text{B}} - b}{V_{\text{A}} - b} \right)^{-R/C_V} = T_{\text{A}} \left(\frac{V_{\text{B}} - b}{V_{\text{A}} - b} \right)^{-2/3} \tag{6.4}$$

(b) Let temperature at A to be T_{A} =100.00 K, $V_{\text{A}} = 8.00\,\text{l}$, $V_{\text{B}} = 1.00\,\text{l}$, $V_{\text{C}} = 2.00\,\text{l}$, $a = 1.355$ $\text{l}^2{\cdot}\text{atm/mol}^2$, and $b = 0.0313\,\text{l/mol}$. Calculate the highest temperature reached during the whole cycle. **[1½]**

Highest temperature =

Solution: Highest temperature during cycle is at T_C.

$$T_{\text{B}} = 407.5\,\text{K} \tag{6.5}$$

which gives, from van der Waal equation

$$P_{\text{B}} = 33.18\,\text{atm} = P_{\text{C}} \tag{6.6}$$

$$\Rightarrow T_{\text{C}} = 803.76\,\text{K} \approx 804\,\text{K} \tag{6.7}$$

Different value of T_C (within a range) obtained due to reasonable roundoff in previous step(s) will be credited.

(c) Calculate the efficiency η of the cycle. **[3]**

Value of η =

Solution:

$$dQ_{\text{in}} = dQ_{\text{BC}} = dU + P_{\text{B}} dV \tag{6.8}$$

$$= C_V\,dT + \frac{RT}{V-b}\,dV \tag{6.9}$$

$$= C_V\,dT + \left(P + \frac{a}{V^2}\right) dV \tag{6.10}$$

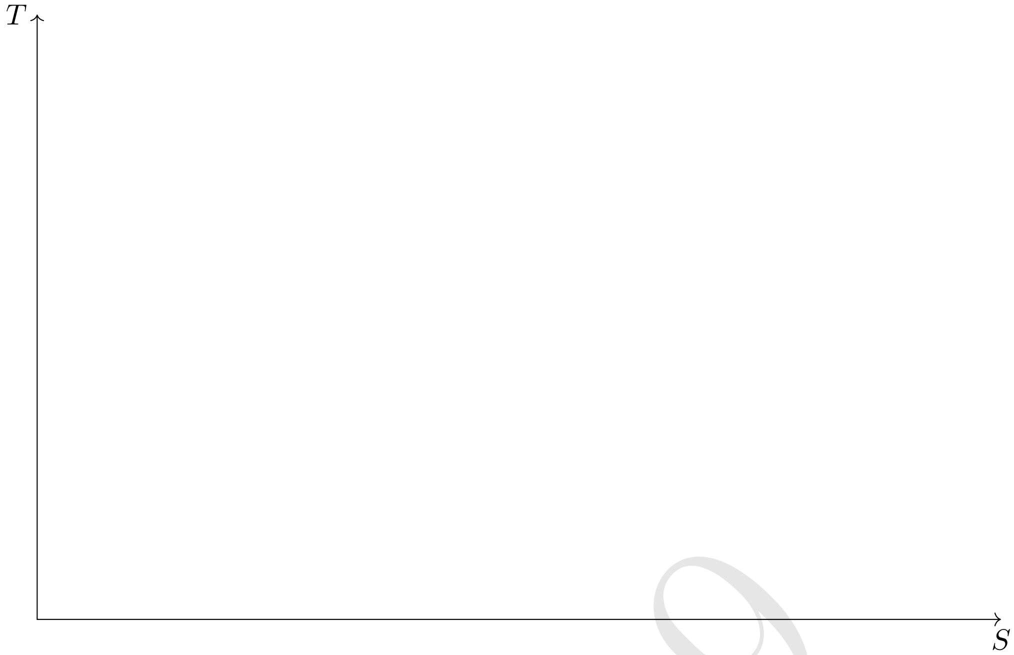
$$Q_{\text{in}} = C_V(T_{\text{C}} - T_{\text{B}}) + P_{\text{B}}(V_{\text{C}} - V_{\text{B}}) - \frac{a}{V}\bigg|_{V_{\text{B}}}^{V_{\text{C}}} \tag{6.11}$$

$$Q_{\text{out}} = Q_{\text{DA}} = C_V(T_{\text{A}} - T_{\text{D}}) \tag{6.12}$$

$$\eta = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 67.7\% \approx 68\% \tag{6.13}$$

Different value of η (within a range) obtained due to reasonable roundoff in previous step(s) will be credited.

(d) Draw the corresponding T-S (entropy) and V-T diagram for the Diesel cycle. Wherever possible, mention the numerical values of T, V, and S on the diagrams. **[7]**

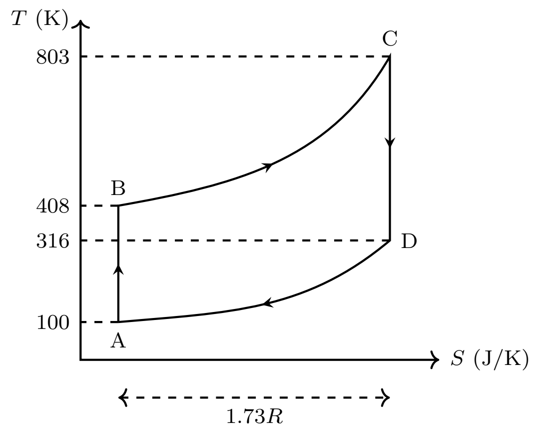


Solution:

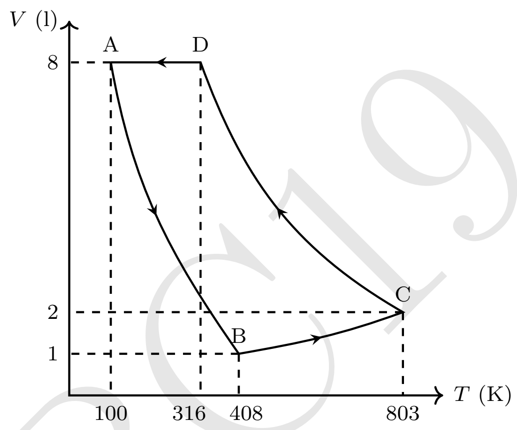
$$\Delta S_{\mathrm{BC}} = \int \frac{dQ}{dT} = \int \frac{C_V\, dT}{T} + \int \frac{R}{V-b} dV \tag{6.14}$$

$$= C_V \ln \frac{T_C}{T_{\mathrm{B}}} + R \ln \frac{V_C - b}{V_{\mathrm{B}} - b} \tag{6.15}$$

$$= 1.73R = \Delta S_{\mathrm{DA}} \tag{6.16}$$



Solution:



Detailed answers can be found on page numbers:

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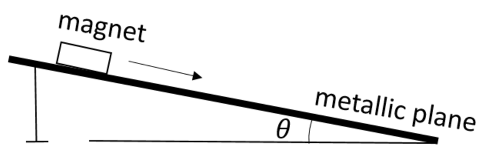
7. As shown, a magnet of mass $m = 19.00$g slides on a rough non-magnetic metallic inclined plane which makes an angle θ with the horizontal. One can change the angle of inclination of the plane. Due to relative motion between the magnet and the plane, eddy currents are generated inside the metal which retard motion of the magnet.



Assume that the magnitude of the resistive force on the magnet is bv where b is a positive constant and v is the instantaneous speed of the magnet. Let μ be the coefficient of kinetic friction between the magnet and the plane.

(a) If the magnet starts moving from rest at time $t = 0$, obtain the expression of its terminal velocity V_{T}. Also obtain the displacement $S(t)$ along the inclined plane as a function of time. Take $S(0) = 0$. **[2]**

$V_{\mathrm{T}} =$

$S(t) =$

Solution:

$$V_T = \frac{mg(\sin\theta - \mu\cos\theta)}{b} \tag{7.1}$$

$$S(t) = V_T\left[t - \frac{m}{b}\left(1 - e^{-tb/m}\right)\right] \tag{7.2}$$

(b) For a fixed θ a student records S for all values of t as shown in the table below. Draw a suitable graph and obtain the terminal velocity V_T of the magnet from this graph. For this and the next part, three graph papers are provided with this booklet. No extra graph papers will be provided. **[4]**

t (s)	S (m)
0.016	0.001
0.049	0.003
0.070	0.006
0.090	0.010
0.120	0.017
0.174	0.029
0.230	0.046
0.270	0.058
0.320	0.074
0.370	0.091

$V_T =$

Graph is plotted on page no. : ______

Solution: V_T = slope of the graph = 0.315 m/s
Accepted range $0.301\,\text{m/s} \le V_T \le 0.329\,\text{m/s}$.

(c) The above process is repeated for various values of θ. The obtained values of terminal velocities for the different θ are given below. **[7]**

θ (degree)	V_T (m/s)
19	0.15
24	0.23
28	0.29
35	0.40
40	0.46
45	0.53
48	0.58
52	0.62

Plot a suitable graph and obtain the values of b and μ from this graph.

$b =$	$\mu =$

Graph is plotted on page no. : ______

Solution:

$$\frac{V_{\mathrm{T}}}{\cos\theta} = \frac{mg}{b}\tan\theta - \frac{\mu mg}{b} \tag{7.3}$$

Graph is plotted for $V_{\mathrm{T}}/\cos\theta$ vs $\tan\theta$.

$\tan\theta$	$V_{\mathrm{T}}/\cos\theta$ (m/s)
0.34	0.16
0.45	0.25
0.53	0.33
0.70	0.49
0.84	0.60
1.00	0.75
1.11	0.87
1.28	1.01

Slope of the graph $= 0.91\,\mathrm{m/s} = \dfrac{mg}{b} \Rightarrow b = 0.21\,\mathrm{N{\cdot}s/m}$

Intercept $= 0.16 = \dfrac{\mu mg}{b} \Rightarrow \mu = 0.18$

Accepted range of slope = 0.86 - 0.96 m/s

Accepted range of intercept = 0.12 - 0.19 m/s

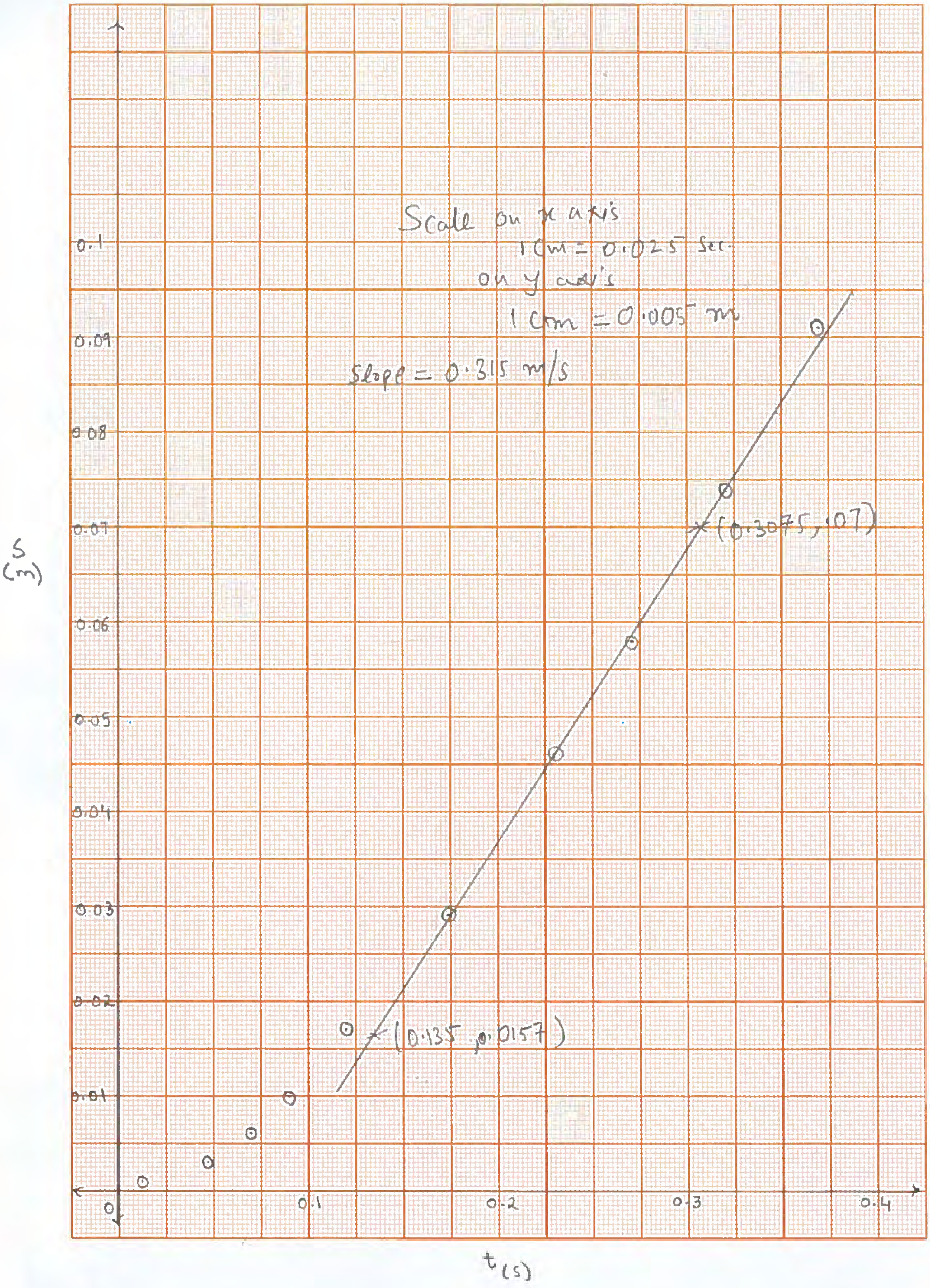
Respective ranges of b and μ are

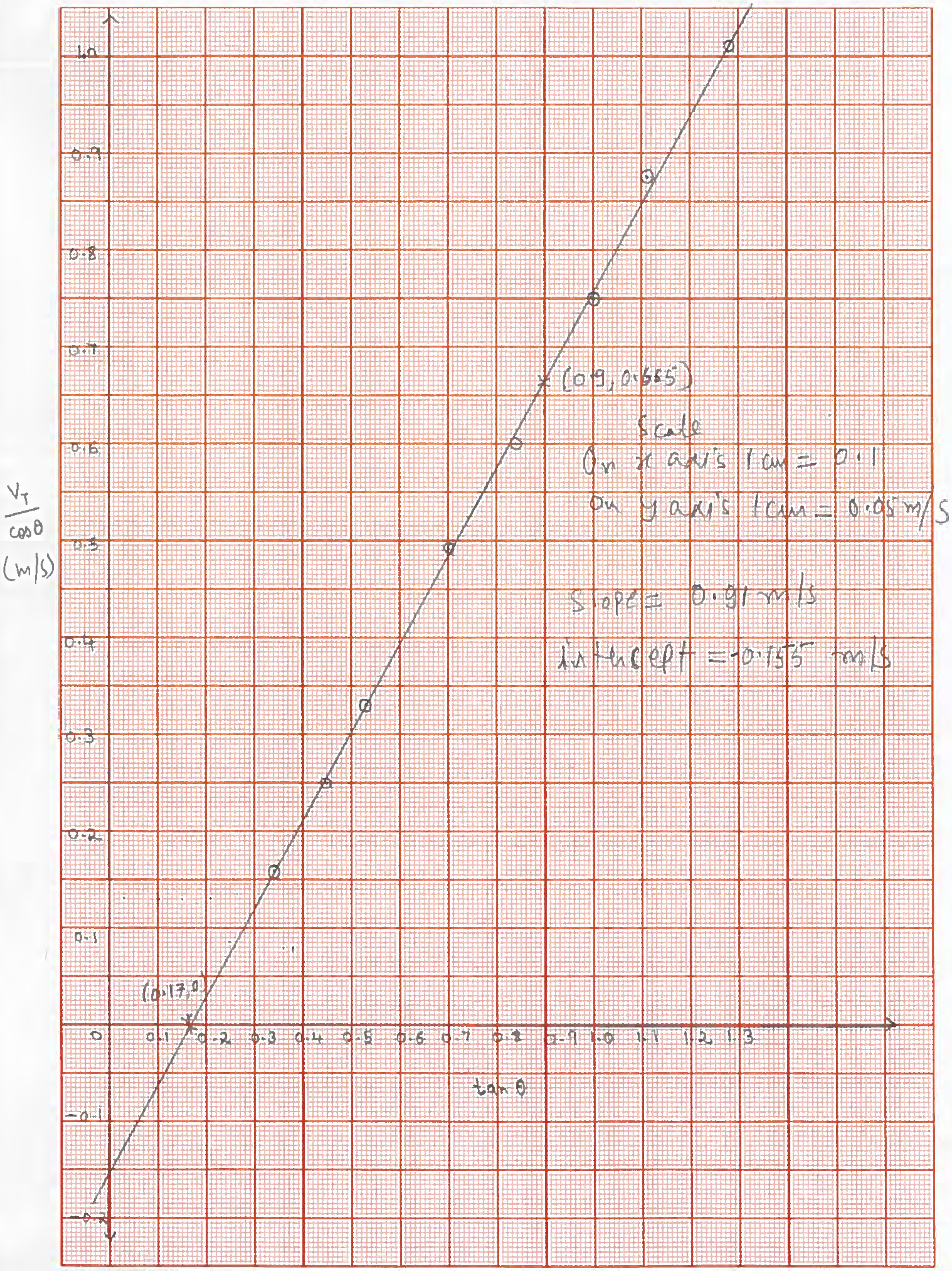
$0.19 \leq b \leq 0.22\,\mathrm{m/s}$ and $0.14 \leq \mu \leq 0.20$

Determination of b and μ from a graph of V_{T} vs θ using an appropriate method also gains credit.

Detailed answers can be found on page numbers:

****** END OF THE QUESTION PAPER ******


Scale on x axis
1 cm = 0.025 sec.
on y axis
1 cm = 0.005 m
Slope = 0.315 m/s
(0.3075, .07)
(0.135, 0.0157)
S (m)
t (s)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.1
0.2
0.3
0.4


$\frac{V_T}{\cos\theta}$ (m/s)
tan θ
(0.9, 0.665)
(0.17, 0)
Scale
On x axis 1 cm = 0.1
On y axis 1 cm = 0.05 m/s
Slope = 0.91 m/s
intercept = −0.155 m/s

Solutions of Indian National Physics Olympiad – 2020

Date: 02 February 2020
Time : **09:00-12:00 (3 hours)**

Roll Number: 2 0 - -
Maximum Marks: **80**

Extra sheets attached :

INO Centre (e.g. Ranchi)

(Do not write below this line)

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Instructions

1. This booklet consists of 16 pages (excluding this page) and total of 5 questions.
2. This booklet is divided in two parts: **Questions with Summary Answer Sheet** and **Detailed Answer Sheet**. Write roll number at the top wherever asked.
3. **The final answer to each sub-question should be neatly written in the box provided below each sub-question in the Questions & Summary Answer Sheet.**
4. You are also required to show your **detailed work** for each question in a reasonably neat and coherent way in the **Detailed Answer Sheet**. You must write the relevant Question Number(s) on each of these pages.
5. Marks will be awarded on the basis of what you write on both the Summary Answer Sheet and the Detailed Answer Sheet. Simple short answers and plots may be directly entered in the Summary Answer Sheet. Marks may be deducted for absence of detailed work in questions involving longer calculations. Strike out any rough work that you do not want to be considered for evaluation.
6. Adequate space has been provided in the answersheet for you to write/calculate your answers. In case you need extra space to write, you may request additional blank sheets (maximum two) from the invigilator. Write your roll number on the extra sheets and get them attached to your answersheet and indicate number of extra sheets attached at the top of this page.
7. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.
8. Use blue or black pen to write answers. Pencil may be used for diagrams/graphs/sketches.
9. **This entire booklet must be returned at the end of the examination.**

Table of Constants

Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Planck's constant	h	6.63×10^{-34} J·s
	$\hbar$	$h/2\pi$
Universal constant of Gravitation	G	6.67×10^{-11} N·m^2·kg^{-2}
Magnitude of electron charge	e	1.60×10^{-19} C
Rest mass of electron	m_e	9.11×10^{-31} kg
Value of $1/4\pi\epsilon_0$		9.00×10^{9} N·m^2·C^{-2}
Avogadro's number	N_A	6.022×10^{23} mol^{-1}
Acceleration due to gravity	g	9.81 m·s^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mol^{-1}
	R	0.0821 l·atm·mol^{-1}·K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H·m^{-1}

Question	Marks	Score
1	13	
2	12	
3	15	
4	20	
5	20	
Total	80	

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

Please note that alternate/equivalent methods and different way of expressing final solutions may exist.

1. A certain gas obeys the equation of state $U(S,V,N) = aS^7/V^4N^2$, where a is a dimensioned constant. Here U represents the internal energy of the gas, S the entropy, V the volume and N the fixed number of particles of the system.

   (a) Let such a gas be filled in a box of volume V and the internal energy of the system be U. A partition is placed to divide the box into two equal parts, each having volume $V/2$. For each part, the internal energy is now αU and the dimensioned constant be βa. Obtain α and β. **[3]**

$\alpha =$	$\beta =$

   Solution: An extensive parameter of the system gets halved if the size of the system is halved, while an intensive parameter remains unchanged. The internal energy and the entropy, both are extensive parameters. Thus $\alpha = 1/2, \quad \beta = 1$.

   (b) The temperature T can be expressed in terms of the derivative of internal energy as **[2]**

   $$T = \left(\frac{dU}{dS}\right)_{V,N}$$

   where the subscripts indicate that the differentiation has been carried out keeping V and N constant. In a similar way, express pressure P in terms of a derivative of the internal energy.

   $P =$

   Solution:

   $$P = -\left(\frac{dU}{dV}\right)_{S,N}$$

   (c) Find the equation of state of the given system relating P, T , and V. **[1]**

   $P =$

   Solution: From the definition of temperature,

   $$T = \frac{7aS^6}{N^2V^4}$$

   From part (b)

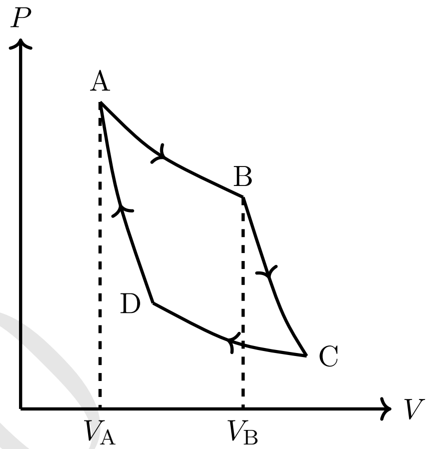
   $$P = \frac{4aS^7}{V^5N^2}$$

Eliminating S yields

$$P = C\frac{T^{7/6}}{V^{1/3}}$$

where $C = \dfrac{4N^{1/3}}{7^{7/6}a^{1/6}}$

(d) One mole of this gas executes a Carnot cycle ABCDA between reservoirs at temperatures T_1 and T_2 ($T_1 > T_2$). Obtain the heat change in the process AB (Q_{AB}) and work done by the system in the processes AB and BC ($W_{\mathrm{AB}}, W_{\mathrm{BC}}$) of the cycle. Express your answers only in terms of temperatures T_1, T_2, volumes V_A, V_B, and the other constants. **[7]**



$Q_{\mathrm{AB}} =$

$W_{\mathrm{AB}} =$

$W_{\mathrm{BC}} =$

Solution:

$$S = \frac{7}{4}\frac{PV}{T}, \quad U = \frac{PV}{4}$$

Leg AB: A $\rightarrow$ B (isothermal)$\Rightarrow$ T_1 = constant

$$Q_{\mathrm{AB}} = T_1 \int_{\mathrm{A}}^{\mathrm{B}} dS \tag{1.1}$$

$$= \frac{7}{4}(P_B V_B - P_A V_A) \tag{1.2}$$

Using equation of state,

$$\boxed{Q_{\mathrm{AB}} = \frac{7C}{4}\left[V_B^{\frac{2}{3}} - V_A^{\frac{2}{3}}\right]T_1^{\frac{7}{6}}}$$

$$W_{\mathrm{AB}} = \int_{\mathrm{A}}^{\mathrm{B}} PdV = c\int_{\mathrm{A}}^{\mathrm{B}} V^{-1/3}T_1^{7/6}dV \tag{1.3}$$

$$\boxed{W_{\mathrm{AB}} = \frac{3C}{2}T_1^{7/6}\left[V_B^{\frac{2}{3}} - V_A^{\frac{2}{3}}\right]}$$

Leg BC is isentropic/adiabatic $\Rightarrow Q = 0$ or $S =$ constant. From first law,

$$W_{\mathrm{BC}} = -\Delta U = \frac{aS^7}{N^2}\left[\frac{1}{V_B^4} - \frac{1}{V_C^4}\right] \tag{1.4}$$

$$= \frac{a}{N^2}\left[\frac{S^7}{V_B^4} - \frac{S^7}{V_C^4}\right] \tag{1.5}$$

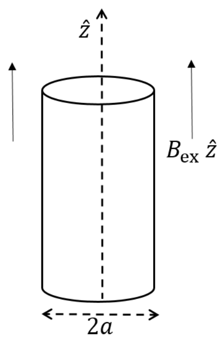
Also, $S = \frac{7C}{4}V^{2/3}T^{1/6}$

$$W_{\mathrm{BC}} = \frac{a}{N^2}\left(\frac{7C}{4}\right)^7\left[V_B^{\frac{2}{3}}T_1^{\frac{7}{6}} - V_C^{\frac{2}{3}}T_2^{\frac{7}{6}}\right] \tag{1.6}$$

$$\boxed{W_{\mathrm{BC}} = \frac{a}{N^2}\left[\frac{7C}{4}\right]^7 V_B^{\frac{2}{3}}T_1^{\frac{1}{6}}[T_1 - T_2]}$$

Detailed answers can be found on page numbers:

2. An insulating uniformly charged cylindrical shell of radius a lies with its axis along the z axis. The shell's moment of inertia per unit length about the z axis and the surface charge density are I and σ respectively. The cylinder is placed in an external uniform magnetic field $B_{\mathrm{ex}}\hat{z}$, and is initially at rest. Starting at $t = 0$ the external magnetic field is slowly reduced to zero. What is the final angular velocity ω of the cylinder? **[12]**



$\omega =$

Detailed answers can be found on page numbers:

Solution: As the magnetic field drops, its time derivative results in an induced electric field, in azimuthal direction.

$$E_\phi(r) = -\frac{r\dot{B}_z}{2} \tag{2.1}$$

This field acts on the charged cylindrical shell to produce azimuthal torque (per unit length).

$$\tau_\phi = aE_\phi(a)2\pi a\sigma \tag{2.2}$$

$$\tau_\phi = I\frac{d\omega}{dt} \tag{2.3}$$

$$d\omega = -\frac{\sigma\pi a^3}{I}dB_z \tag{2.4}$$

$$\omega = -\frac{\sigma\pi a^3}{I}\int_{B_i}^{B_f} dB_z \tag{2.5}$$

$$= \frac{\sigma\pi a^3}{I}(B_i - B_f) \tag{2.6}$$

$B_i = B_{\text{ex}}$ and B_f is the non-zero magnetic field produced by the rotating charged cylinder. For the rotating cylinder, compare with solenoid,the magnetic field will be along z axis and equal to

$$\vec{B_f} = \mu_0 ni\hat{z} \tag{2.7}$$

$$= \mu_0 j_\phi \hat{z} \tag{2.8}$$

where $j_\phi = a\sigma\omega$ is azimuthal current per unit length. Thus

$$\vec{B_f} = \mu_0 a\sigma\omega\hat{z}$$

$$\omega = \frac{\sigma\pi a^3}{I}\left(B_{\text{ex}} - \mu_0 a\sigma\omega\right) \tag{2.9}$$

$$\omega = \frac{\sigma\pi a^3}{I}\left(1 + \frac{\mu_0\sigma^2 a^4\pi}{I}\right)^{-1} B_{\text{ex}} \tag{2.10}$$

3. Consider the Bohr model of the hydrogen atom. Let m_e and e be the mass and magnitude of the charge of the electron respectively. Let a_0 be the ground state radius (Bohr radius).

   (a) Obtain an expression for the ionisation energy I_{H} of the ground state of the hydrogen atom in terms of a_0 and constants. **[1]**

   $I_{\text{H}} =$

Solution: Centripetal acceleration is given by Coulomb force.

$$\frac{m_e v^2}{a_0} = \frac{Ke^2}{a_0^2}; \quad K = \frac{1}{4\pi\epsilon_0}; \quad m_e v r = \hbar \tag{3.1}$$

$$a_0 = \frac{\hbar^2}{K m_e e^2} \tag{3.2}$$

$$\text{Total energy} = \frac{m_e v^2}{2} - \frac{Ke^2}{a_0} = -\frac{Ke^2}{2a_0} \tag{3.3}$$

$$I_{\mathrm{H}} = \frac{Ke^2}{2a_0} \tag{3.4}$$

(b) Consider a singly ionised helium atom He^+. Obtain the ground state ionisation energy I_{He^+} of He^+ in terms of I_{H}. **[2]**

$I_{\mathrm{He}^+} =$

Solution:

$$\frac{m_e v^2}{r} = \frac{2Ke^2}{r^2} \text{ and } m_e v r = \hbar \tag{3.5}$$

$$r = \frac{\hbar^2}{2Kme^2} = \frac{a_0}{2} \tag{3.6}$$

$$\text{Total Energy (T.E.)} = \frac{1}{2} m_e v^2 - \frac{2Ke^2}{r} = -\frac{Ke^2}{r} = \frac{2Ke^2}{a_0} = -4I_H \tag{3.7}$$

$$I_{\mathrm{He}^+} = 4I_{\mathrm{H}} \tag{3.8}$$

(c) Now consider a two electron system with arbitrary atomic number Z. Use the Bohr model to obtain the ground state radius $(r(Z))$ in terms of a_0 and Z. Assume the two electrons are in the same circular orbit and as far apart as possible. **[1]**

$r(Z) =$

Solution:

$$\frac{m_e v^2}{r} = \frac{KZe^2}{r^2} - \frac{Ke^2}{(2r)^2} \text{ and } m_e v r = \hbar \tag{3.9}$$

$$m_e v^2 r = Ke^2 \left(Z - \frac{1}{4}\right) \text{ and } m_e v^2 r = \frac{\hbar^2}{m_e r} \tag{3.10}$$

$$r = \frac{\hbar^2}{K m_e e^2 \left(Z - \frac{1}{4}\right)} = \frac{a_0}{\left(Z - \frac{1}{4}\right)} \tag{3.11}$$

(d) Derive an expression for the first ionisation energy I_Z^{th} for two electron system with arbitrary Z in terms of Z and I_{H}. **[3]**

$I_Z =$

Solution:

$$\text{Kinetic Energy (K.E.)} = m_e v^2 = \frac{Ke^2\left(Z - \frac{1}{4}\right)}{r} \tag{3.12}$$

$$= \frac{Ke^2\left(Z - \frac{1}{4}\right)^2}{a_0} \tag{3.13}$$

$$= 2\left(Z - \frac{1}{4}\right)^2 I_{\text{H}} \tag{3.14}$$

$$\text{Potential Energy (P.E.)} = -\frac{2KZe^2}{r} + \frac{Ke^2}{2r} \tag{3.15}$$

$$= \frac{-2Ke^2}{r}\left(Z - \frac{1}{4}\right) \tag{3.16}$$

$$= \frac{-2Ke^2}{a_0}\left(Z - \frac{1}{4}\right)^2 \tag{3.17}$$

$$= -4\left(Z - \frac{1}{4}\right)^2 I_{\text{H}} \tag{3.18}$$

$$(\text{T.E.})_{\text{i}} = -2\left(Z - \frac{1}{4}\right)^2 I_{\text{H}} \tag{3.19}$$

$$(\text{T.E.})_{\text{f}} = -Z^2 I_{\text{H}} \tag{3.20}$$

$$I_Z^{\text{th}} = (\text{T.E})_{\text{f}} - (\text{T.E})_{\text{i}} = 2\left(Z - \frac{1}{4}\right)^2 I_{\text{H}} - Z^2 I_{\text{H}} \tag{3.21}$$

$$I_Z^{\text{th}} = \left(Z^2 - Z + \frac{1}{8}\right) I_H \tag{3.22}$$

(e) The table below contains the experimental data of I_Z^{expt} (in units of Rydberg where 1 Ryd = 13.6 eV) versus Z for various two-electron systems. **[8]**

	Z	I_Z^{expt}
H^-	1	0.055
He	2	1.81
Li^+	3	5.56
Be^{++}	4	11.32
B^{3+}	5	19.07
C^{4+}	6	28.83
N^{5+}	7	40.60
O^{6+}	8	54.37
F^{7+}	9	70.15

Experimental values were not found to be equal to the theoretical predictions. This difference arises mainly from non-inclusion of Pauli's principle in the theoretical derivation of part (d). It was suggested that if the value of Z was reduced by some fixed amount α ($Z^* = Z - \alpha$)

in the final expression of I_Z^{th} obtained in part (d), then $I_{Z^*}^{\mathrm{th}} \approx I_Z^{\mathrm{expt}}$. Draw a suitable linear plot and from the graph find α. Two graph papers are provided with this booklet in case you make a mistake.

$\alpha =$

Solution:

$$\Delta I_Z = I_Z^{\mathrm{th}} - I_Z^{\mathrm{expt}} = I_Z^{\mathrm{th}} - I_{Z^*}^{\mathrm{th}} = \left\{\left[Z^2 - Z + \frac{1}{8}\right] - \left[(Z-\alpha)^2 - (Z-\alpha) + \frac{1}{8}\right]\right\} I_{\mathrm{H}} \tag{3.23}$$

$$\Delta I_Z = \left[-\alpha^2 + 2Z\alpha - \alpha\right] I_{\mathrm{H}} \tag{3.24}$$

	Z	(I_Z^{expt})	I_Z^{th}	$(I_Z^{\mathrm{th}} - I_Z^{\mathrm{expt}})$
H^-	1	0.055	0.125	0.07
He	2	1.81	2.13	0.32
Li^+	3	5.56	6.13	0.57
Be^{++}	4	11.32	12.13	0.81
B^{3+}	5	19.07	20.13	1.06
C^{4+}	6	28.83	30.13	1.30
N^{5+}	7	40.60	42.13	1.53
O^{6+}	8	54.37	56.13	1.76
F^{7+}	9	70.15	72.13	1.98

A plot of ΔI_Z vs Z is a linear graph.
Slope of the graph $=2\alpha = 0.24$
$\alpha = 0.12$
One can also linearize in the following way:

$$I_Z^{\mathrm{expt}} = I_{Z^*}^{\mathrm{th}} = \left(Z - \alpha - \frac{1}{2}\right)^2 - \frac{1}{8} \tag{3.25}$$

$$\sqrt{I_Z^{\mathrm{expt}} + \frac{1}{8}} = Z - \alpha - \frac{1}{2} \tag{3.26}$$

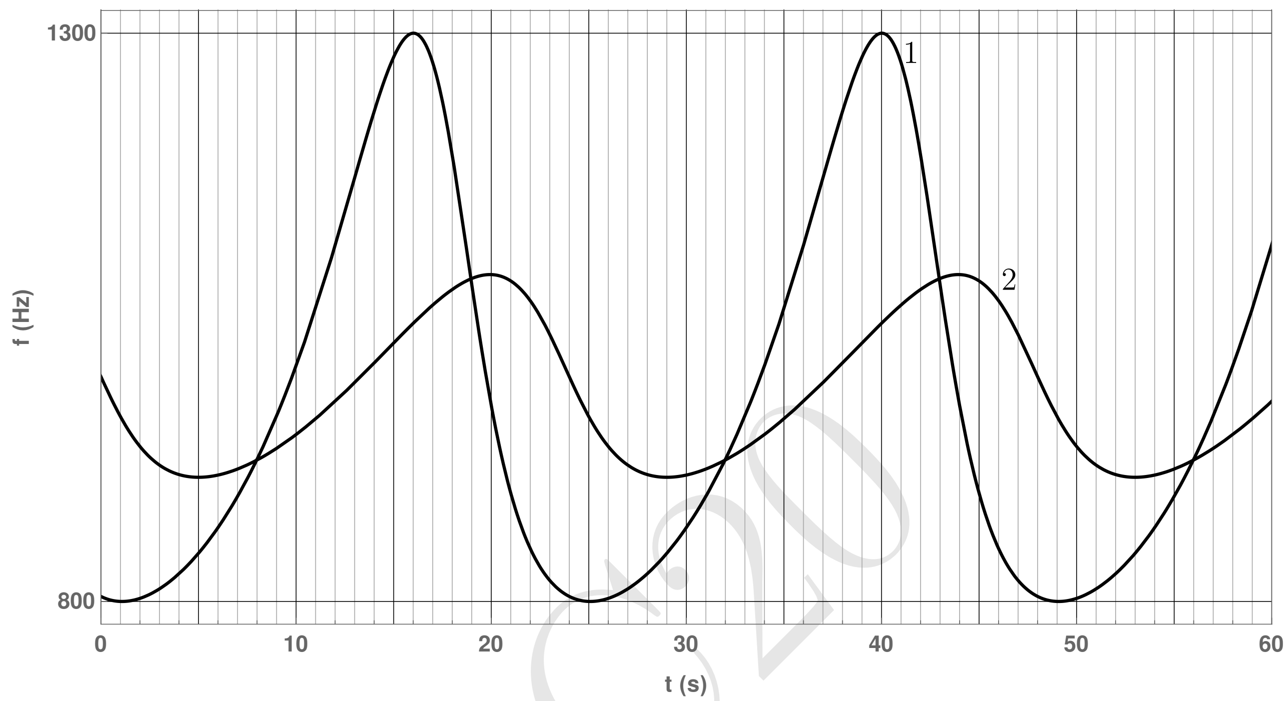
Plot of $\sqrt{I_Z^{\mathrm{expt}} + \frac{1}{8}}$ vs Z is a linear graph. This method gives $\alpha = 0.10$. Both methods are acceptable.

Accepted range of α is : $0.10 \le \alpha \le 0.14$.

Graph is on the last page of this booklet.

Detailed answers can be found on page numbers:

4. A sound source S is performing uniform circular motion with time period T. It is continuously emitting sound of a fixed frequency f_0. Two detectors 1 and 2 are placed somewhere in the same plane as the circular trajectory of the source. The frequency f, of the sound received by the two detectors is plotted as a function of time t as shown below (the clocks of the two detectors are synchronized).



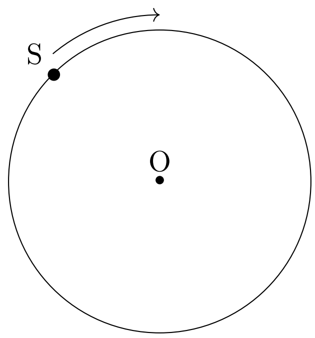
Take the speed of sound in the medium to be 330 m/s.

(a) Determine the time period T of the source. **[2]**

$T =$

Solution: Note the time at the peak frequencies of any of the detectors. For detector 1, first peak is at $t = 16$ s and the second peak is at $t = 40$ s. Hence the time period of the source $T =$24 s.

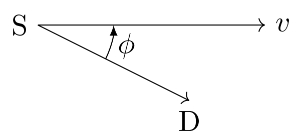
(b) The figure below shows the circular trajectory of the source S. Qualitatively mark the positions of both the detectors by indicating 1 and 2. Here O denotes the centre of the trajectory. You must provide detailed justification of your answer in the detailed answer sheet. **[6]**



Solution: Couple of things can be easily seen from the graphs:
Time period of the source $T = 24\,\mathrm{s}$.
Time difference between the maximum and minimum frequencies detected by detector 1 $= 9\,\mathrm{s}$.
Time difference between the maximum and minimum frequencies detected by detector 2 $= 9\,\mathrm{s}$.
Time difference between the maximum frequencies detected by detector 1 and 2 $= 4\,\mathrm{s}$.

Maximum frequency detected by 1 $= 1300\,\mathrm{Hz}$.
Minimum frequency detected by 1 $= 800\,\mathrm{Hz}$.
Resolution of the graph is not enough to give the maximum and minimum frequencies detected by the detector 2. Let the source S is moving in the circle of radius R. There are four possibilities:

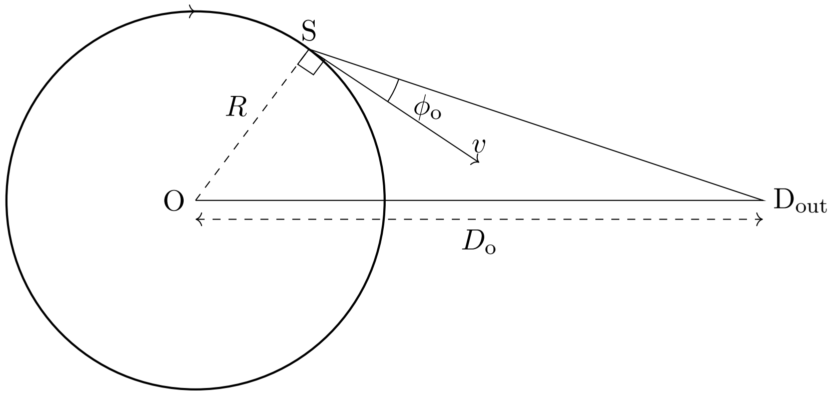
1. Both detectors are outside the circle.
2. Both detectors are inside the circle.
3. One detector is inside and other is outside the circle.
4. One detector is either inside or outside the circle, and other detector is at R.



When source is approaching the stationary detector D is at an angle ϕ, the frequency detected by D is

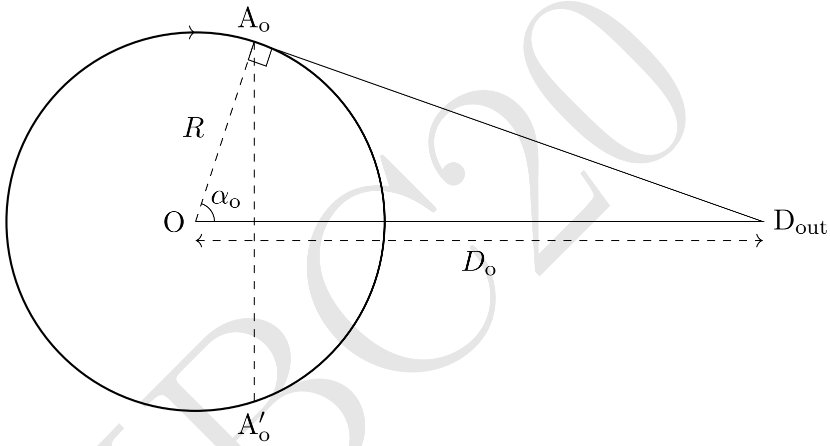
$$f = \frac{f_0}{1 - \frac{v}{c}\cos\phi}$$

Here f_0 is the frequency emitted by the source, and c is the speed of the sound. If we consider fourth possibility, we should have observed a sharp change in the plot when source is crossing the detector. Hence, we rule out this case.
When the detector D_{out} is outside the circle (see figure below):



$$f^{out} = \frac{f_0}{1 - \frac{v}{c}\cos\phi_o}$$

For maximum and minimum frequency, $\cos\phi = \pm 1$

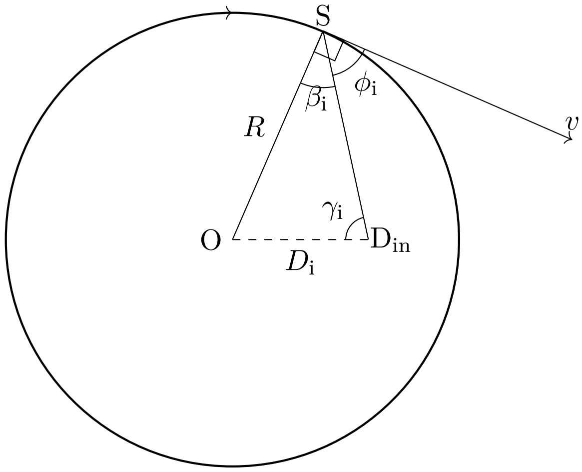


$$\text{at } A_o,\ f^{out}_{max} = \frac{f_0}{1 - \frac{v}{c}} = \frac{f_0}{1 - \frac{\omega R}{c}} \tag{4.1}$$

$$\text{at } A'_o,\ f^{out}_{min} = \frac{f_0}{1 + \frac{v}{c}} = \frac{f_0}{1 + \frac{\omega R}{c}} \tag{4.2}$$

If both the detectors are outside, maximum and minimum frequencies detected by them are same which is not the case if you observe the graph given in the question. Hence, we rule out the first possibility.

When detector D_{in} is inside the circle (see figure below):



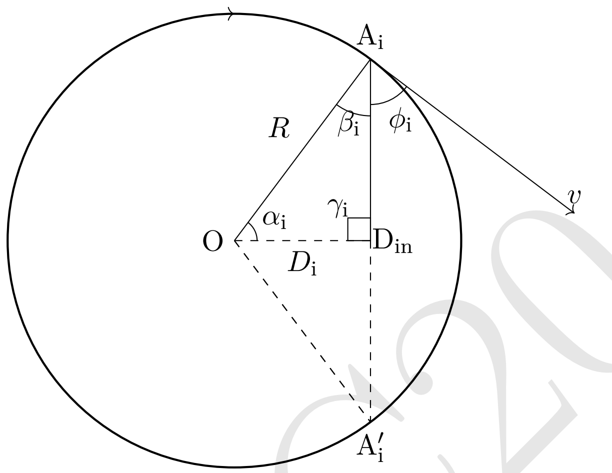
$$f^{\text{in}} = \frac{f_0}{1 - \frac{v}{c}\cos\phi_{\text{i}}} \tag{4.3}$$

$$|\cos\phi_{\text{i}}| = |\sin\beta_{\text{i}}| \tag{4.4}$$

$$\frac{\sin\gamma_{\text{i}}}{R} = \frac{\sin\beta_{\text{i}}}{D_{\text{i}}} \tag{4.5}$$

$$\cos\phi_{\text{i}} = \frac{D_{\text{i}}}{R}\sin\gamma_{\text{i}} \tag{4.6}$$

$$f^{\text{in}} = \frac{f_0}{1 - \frac{v}{c}\frac{D_{\text{i}}}{R}\sin\gamma_{\text{i}}} \tag{4.7}$$



f^{in} is maximum (minimum) when $\sin\gamma_{\text{i}}$ is 1 (-1).

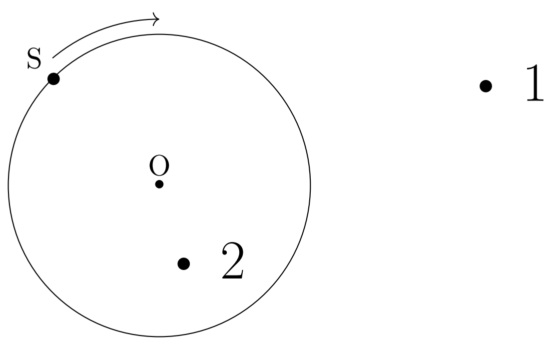
$$\text{at A}_{\text{i}},\ f^{\text{in}}_{\text{max}} = \frac{f_0}{1 - \frac{\omega D_{\text{i}}}{c}} < f^{\text{out}}_{\text{max}} \tag{4.8}$$

$$\text{at A}'_{\text{i}},\ f^{\text{in}}_{\text{min}} = \frac{f_0}{1 + \frac{\omega D_{\text{i}}}{c}} > f^{\text{out}}_{\text{min}} \tag{4.9}$$

If both the detectors are inside, again there are two possibilities: they are at the same distances or at the different distances from the center. In the former case, their observed peak frequencies will be same which is clearly not evident from the given graph. In the latter case, their maximum frequencies will be different but the time difference between the maximum and minimum frequencies will also be different. However from the graph, the time difference between maximum and minimum frequencies for detectors 1 and 2 is

$$t(f_{\text{min}}) - t(f_{\text{max}}) = 9\text{ s}$$

This is possible only if one detector is inside and other detector is outside. Also, detector may not be colinear with the center.



In this part, marking (anywhere) 1 outside and 2 inside the circle with a correct justification will be given full credit.

(c) Obtain the frequency f_0 of the source. **[3]**

$f_0 =$

Solution: From equations (4.1) and (4.2),

$$\frac{f_0}{1-\frac{v}{c}} = 1300 \text{ Hz} \tag{4.10}$$

$$\frac{f_0}{1+\frac{v}{c}} = 800 \text{ Hz} \tag{4.11}$$

$$f_0 \approx 991 \text{ Hz} \tag{4.12}$$

(d) Calculate the distance (D) between the detectors. **[9]**

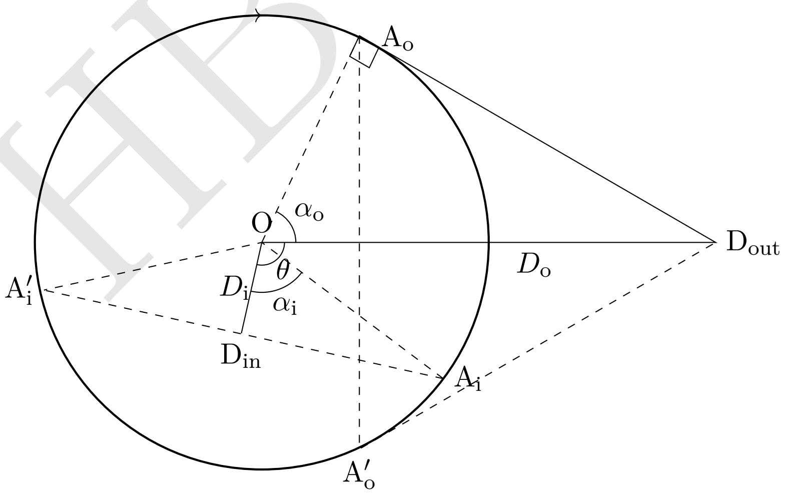
$D =$

Solution: We have to find out D_i, D_o and angle θ between the detectors. From equations (4.10) and (4.11),

$$v = 78.6 \text{ m/s} \tag{4.13}$$

$$v = R\omega = R\frac{2\pi}{24} \tag{4.14}$$

$$R \approx 300 \text{ m} \tag{4.15}$$



By symmetry, $\mathrm{A_o}$ and $\mathrm{A_o'}$ are equidistant from the detector 1. Similarly, $\mathrm{A_i}$ and $\mathrm{A_i'}$ are equidistant from the detector 2. It takes 9 s from $\mathrm{A_i}$ to $\mathrm{A_i'}$ ($\mathrm{A_o}$ and $\mathrm{A_o'}$). Hence

$$2\angle\alpha_\mathrm{o} = 2\angle\alpha_\mathrm{i} = \frac{3\pi}{4} \tag{4.16}$$

$$D_\mathrm{i} = R\cos\alpha_\mathrm{i} \approx 115 \text{ m} \tag{4.17}$$

$$D_\mathrm{o} = \frac{R}{\cos\alpha_\mathrm{o}} \approx 784 \text{ m} \tag{4.18}$$

There will be signal delay due to finite D_o and D_i.
Let θ be the angular separation between the detectors. Time difference between the peak

frequencies of detectors 1 and 2 is 4 seconds.

$$4 = \frac{\theta}{\omega} + \frac{\mathrm{A_i D_{in}}}{c} - \frac{\mathrm{A_o D_{out}}}{c} \tag{4.19}$$

$$4 = \frac{\theta}{\omega} + \frac{R\sin\alpha_\mathrm{i}}{c} - \frac{D_\mathrm{o}\sin\alpha_\mathrm{o}}{c} \tag{4.20}$$

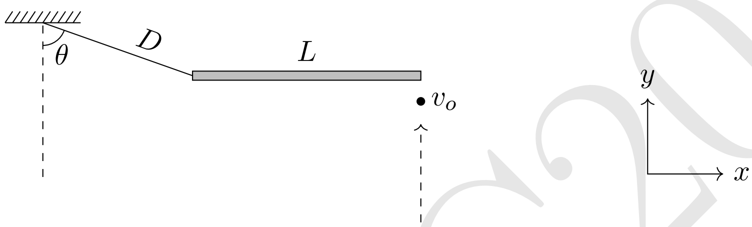
$$\Rightarrow \theta = \frac{\pi}{3} - \frac{\omega R\sin\alpha_\mathrm{i}}{c} + \frac{\omega D_\mathrm{o}\sin\alpha_\mathrm{o}}{c} \tag{4.21}$$

$$= 1.4\,\mathrm{rad} \tag{4.22}$$

The distance between the detectors $= \sqrt{\mathrm{OD_{in}^2} + \mathrm{OD_{out}^2} - 2\mathrm{OD_{in}}.\mathrm{OD_{out}}.\cos(1.4)} \approx 773$ m

Detailed answers can be found on page numbers:

5. The following is the top view of an assembly kept on a smooth horizontal table. **[20]**



A massless inextensible string of length D lies with one end fixed, while the other is attached to one end of a uniform rod of length L. The system is initially at rest with the rod aligned along the x-axis and the string stretched to its natural length at an angle with the negative y-axis θ ($\cos\theta = 1/3$). At a certain instant, a bullet of the same mass m as the rod and negligible dimensions is fired horizontally along the positive y-direction. The bullet hits the rod at its right end with velocity v_o and gets lodged in it, the impact being nearly instantaneous. What is the tension (T) in the string immediately after the impact? Assume the string doesn't break.

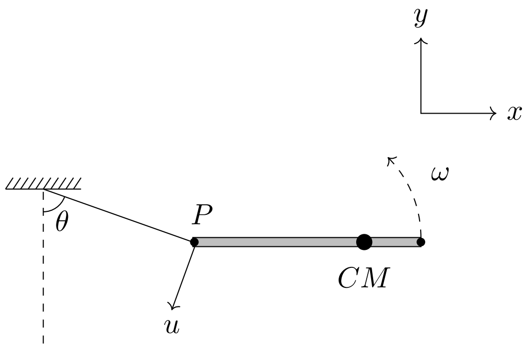
$T =$

Solution: Geometry of the problem dictates that the string becomes taut during the impact and hence exerts an impulse. We conclude:

1. Angular momentum is conserved only about P.

2. The velocity of P is perpendicular to the string right after the impact (refer to the figure).

We have two unknowns - ω and u.



We first note that the momentum of CM is preserved perpendicular to the string since the string applies force only along its length.

$$(P_i)_\perp = (P_f)_\perp$$
$$mv\sin\theta = 2m(v_f)_\perp$$

where $(v_f)_\perp$ is the velocity of the CM perpendicular to the string. CM is located at a distance $\frac{3L}{4}$ from P. So, $(v_f)_\perp = \frac{3L\omega}{4}\sin\theta - u$

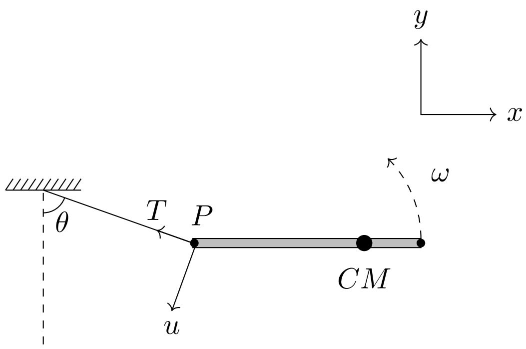
$$\implies \frac{3L\omega}{4}\sin\theta - u = \frac{v_o\sin\theta}{2} \tag{5.1}$$

Applying conservation of angular momentum about P,

$$L_i = L_f$$
$$mv_oL = 2m\Big(\frac{3L\omega}{4} - u\sin\theta\Big)\frac{3L}{4} + I_{\text{cm}}\omega \tag{5.2}$$

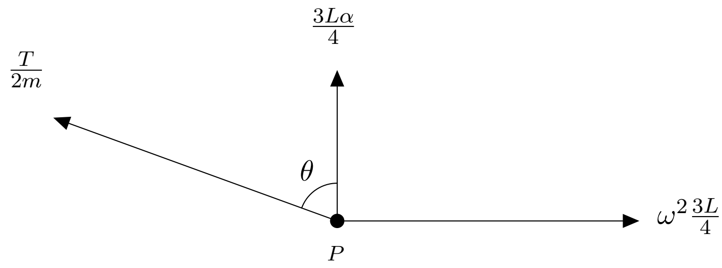
Solving equations 5.1 and 5.2, we get

$$\omega = \frac{v_o}{L} \tag{5.3}$$
$$u = \frac{v_o}{3\sqrt{2}} \tag{5.4}$$



Designate the tension in the string immediately after the impact as T. This gives a clockwise angular acceleration of the rod equal to

$$\alpha = \frac{3L}{4}\frac{T\cos\theta}{I_{\text{cm}}} = \frac{18T\cos\theta}{5mL} \tag{5.5}$$



We now calculate the acceleration of P on the rod in the direction of the string, $(a_P)_{||}$ (in the ground frame) and set it equal to $\frac{u^2}{D}$.

To this end, note that the acceleration of the CM in the direction of the string is equal to $\frac{T}{2m}$. Also, point P is instantaneously rotating about the CM with angular velocity ω in a circle of radius $\frac{3L}{4}$ - this gives a centripetal acceleration of $\omega^2 \times \frac{3L}{4}$ directed towards the CM. Furthermore, the tangential acceleration of P along its trajectory is given by $\frac{3L\alpha}{4}$. Accounting for all these contributions and taking appropriate components, we can write the acceleration of point P in the direction of the string as

$$(a_P)_{||} = \frac{T}{2m} + \frac{27T}{10m}\cos^2\theta - \frac{3\omega^2 L}{4}\sin\theta$$

Now we know

$$(a_P)_{||} = \frac{T}{2m} + \frac{27T}{10m}\cos^2\theta - \frac{3\omega^2 L}{4}\sin\theta = \frac{u^2}{D} \tag{5.6}$$

Hence we get

$$\boxed{T = \frac{5mv_o^2}{4}\left[\frac{1}{18D} + \frac{1}{L\sqrt{2}}\right]}$$

Comment [not for grading purposes]:

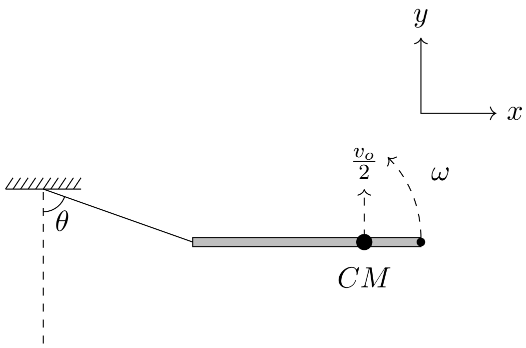
We assumed above that the string is taut. This is consistent with the solution above, as we found a non-zero answer for the tension. However, if you do not picture that the geometry of the problem requires tension in the string, you can convince yourselves that this is the correct assumption, by assuming that the tension is zero and arriving at a contradiction.

Assume that the string becomes slack. Let the angular velocity of the rod be ω counter-clockwise, as seen from top, immediately after the impact. Then, noting that velocity of CM remains preserved (because the string is assumed to go slack during the impact) and applying conservation of angular momentum about the point of impact, one gets

$$\begin{aligned} L_i &= L_f \\ 0 &= 2m\frac{v_o}{2}\frac{L}{4} - I_{\text{cm}}\omega \\ \omega &= \frac{mv_oL}{4I_{\text{cm}}} \end{aligned}$$

where I_{cm} is the moment of inertia of the rod and the bullet system about its center of mass. Now, $I_{\text{cm}} = \frac{mL^2}{12} + \frac{mL^2}{8} + \frac{mL^2}{8} = \frac{5mL^2}{24}$, thereby yielding

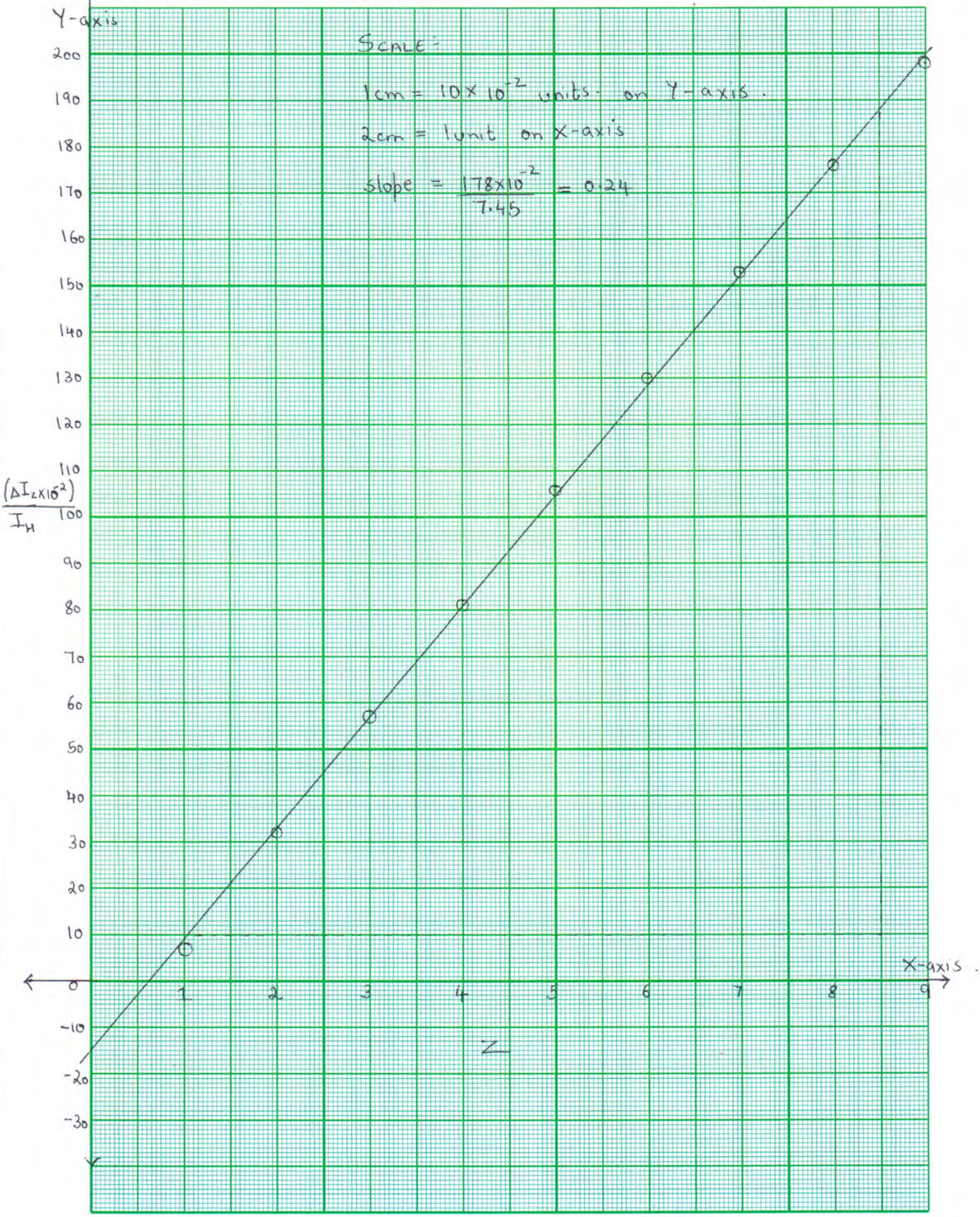
$$\omega = \frac{6v_o}{5L}$$



The velocity of the end of the string tied to the rod (P) immediately after the impact, therefore, would be given by $-\left(\frac{3L}{4} \times \frac{6v_o}{5L} - \frac{v_o}{2}\right)\hat{\jmath} = -\frac{2v_o}{5}\hat{\jmath}$. But this would mean the string would get elongated. Contradiction!

Detailed answers can be found on page numbers:

****** END OF THE QUESTION PAPER ******


Y-axis
SCALE:
1cm = 10 × 10⁻² units on Y-axis.
2cm = 1unit on X-axis
slope = 178×10⁻² / 7.45 = 0.24
200
190
180
170
160
150
140
130
120
110
(ΔI₂×10⁻²) / I_H
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
1
2
3
4
5
6
7
8
9
X-axis
Z

Indian National Physics Olympiad (INPhO)-2023

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
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Solutions

Date: 29 January 2023
Time: **09:00-12:00 (3 hours)** Maximum Marks: **60**

Instructions **Roll No.:**

1. This booklet consists of 19 pages and total of 5 questions. Write roll number at the top wherever asked.

2. Booklet to write the answers is provided separately. Instructions to write the answers are on the Answer Booklet.

3. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.

4. **Please submit the Answer Sheet at the end of the examination.** You may retain the Question Paper.

Table of Constants

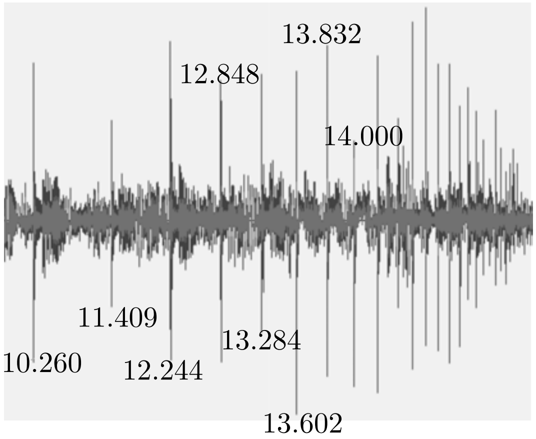
Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Planck's constant	h	6.63×10^{-34} J·s
	$\hbar$	$h/2\pi$
Universal constant of Gravitation	G	6.67×10^{-11} N·m^2·kg^{-2}
Magnitude of electron charge	e	1.60×10^{-19} C
Rest mass of electron	m_e	9.11×10^{-31} kg
Value of $1/4\pi\epsilon_0$		9.00×10^{9} N·m^2·C^{-2}
Avogadro's number	N_A	6.022×10^{23} mol^{-1}
Acceleration due to gravity	g	9.81 m·s^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mol^{-1}
	R	0.0821 l·atm·mol^{-1}·K^{-1}
Boltzmann constant	K_B	1.3806×10^{-23} J·K^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H·m^{-1}
1 Angstrom unit	1Å	1×10^{-10} m
1 micro unit	1μ	1×10^{-6}units
1 electron volt	1 eV	1.6×10^{-19} J

Q No	**1**	**2**	**3**	**4**	**5**	**Total**
Maximum Marks	6	6	16	16	16	60

Please note that alternate/equivalent methods and different way of expressing final solutions may exist. A correct method will be suitably awarded.

1. **[6 marks] Dancing on the floor**

There are various apps that record the intensity of an audio signal. An app (WaveEditor$^{\mathrm{TM}}$ here) displays the audio signal as a wave, whose amplitude is proportional to the audio signal's loudness. A smartphone with this app recording the sound signal is kept on a uniformly built flat floor of a classroom.



A perfectly small spherical steel ball is thrown up such that it almost touches the ceiling and comes back without hitting. The ball hits the floor and thereafter it keeps bouncing. The app records the sound signal produced when the ball hits the floor on every bounce. A screenshot of the recording is shown. The timestamps (in seconds) of the first eight consecutive bounces are also shown next to the peak. For example, the app records a peak at 10.260 s when the first time the ball hits the floor.

Make reasonable assumptions, when the ball hits the floor and calculate the height of the classroom from the given data. State your assumptions clearly.

Solution: The initial height of the peaks seems to be random. This might happen when the ball hits the floor near the phone and some time away from the phone. The time interval between the peaks is reducing, indicating that the ball is colliding inelastically with the floor.

It is also not given when the app started recording the sound. If we take the timestamp of first peak (10.26 s) to be the true time taken for the first bounce, that will give the height of the room to be 131 m which is a nonphysical number for a classroom's height.

Since the ball and the floor both are uniformly shaped objects, we can consider that in each bounce, the ball loses the same amount of energy. Let the height of the room be h_0. The ball attains the height h_1, and h_2 after the first and the second bounce respectively.

$$\frac{E_0}{E_1} = \frac{E_1}{E_2} = \frac{E_2}{E_3} = ... \tag{1.1}$$

$$\Rightarrow \frac{h_0}{h_1} = \frac{h_1}{h_2} \tag{1.2}$$

Let the time interval between the first and the second bounce be Δt_1 and the time interval between second and the third bounce be and Δt_2. This yields

$$h_0 = \frac{h_1^2}{h_2} \tag{1.3}$$

where

$$h_1 = \frac{1}{2} g \left(\frac{\Delta t_1}{2}\right)^2 \tag{1.4}$$

$$h_2 = \frac{1}{2} g \left(\frac{\Delta t_2}{2}\right)^2 \tag{1.5}$$

Substituting Eqs. (1.4) and (1.5) in Eq.(1.3), we get

$$h_0 = \frac{g}{8}\frac{\Delta t_1^4}{\Delta t_2^2} = 3.06\,\mathrm{m} \tag{1.6}$$

Alternate ways of solving exist. The accepted range of h_0 is 2.80m − 3.10m.

2. **Knock it off!**

Consider a 100 W small isotropic source of blue light of wavelength 4500Å. A metallic surface of 1.00 cm^2 and work function 2.20 eV is kept at a distance of 1.00 m from the source and oriented to receive normal radiation.

(a) [**2 marks**] Assume that all the energy is uniformly absorbed by atoms on the top layer of the surface. Also, all the energy absorbed by an atom on the surface is taken up by one electron. The radius of the atom is 1.00Å. Estimate the time τ_{e} needed by the electron to receive 1.00 eV of energy.

Solution:

$$\frac{P}{4\pi R^2}\pi \times r^2 \Delta t = 1\mathrm{eV} \tag{2.1}$$

$$\Delta t = \tau = 0.64\,\mathrm{sec} \tag{2.2}$$

(b) [**1 marks**] According to the above classical model, how many electrons are emitted by the metallic surface in time τ_{e}?

Solution: None, since the work function is 2.2eV. The electron needs another 1.41sec.

(c) [**2 marks**] In quantum theory, photons are emitted and absorbed as quanta. Assuming photoelectric efficiency of 1%, calculate the rate of emission of electrons (N_{e}) from the surface.

Solution: $N_e = 1.8 \times 10^{13}\,\mathrm{s}^{-1}$

(d) [**1 marks**] Assuming further that all the emitted photoelectrons move normal to the surface what would be the maximum current density ($J_{\max}$) one may expect?

Solution:

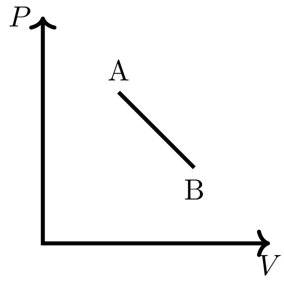
$$\text{Current density } J = \frac{N_e e}{A}$$

where A is the area of metallic surface.

$$J_{\max} = 2.88 \times 10^{-2}\,\mathrm{Amp/m^2} \tag{2.3}$$

3. **[16 marks] Work in progress**

One mole of an ideal monoatomic gas goes through a linear process from A to B as shown in the pressure-volume (P-V) diagram. The temperature at A is $T_{\text{A}} = 227°\text{C}$. The process is such that, the temperature decreases and the heat is continuously supplied to the gas. The ratio of the specific heat at the constant pressure to that at the constant volume is 5/3. Obtain the expression for the maximum work (W_{max}) the gas can perform in such a process. Calculate W_{max}.



Solution: The variation of P is linear with respect to V, hence it can be written as

$$P = -aV + b \tag{3.1}$$

where a and b are positive constants. At A,

$$P_A V_A = RT_A \tag{3.2}$$

$$(-aV_A + b)V_A = R \times 500 \tag{3.3}$$

where $T_A = 500\,\text{K}$. Using the ideal gas equation $PV = RT$

$$T = \frac{PV}{R} = \frac{-aV^2 + bV}{R} \tag{3.4}$$

$$\frac{dT}{dV} = \frac{-2aV + b}{R} \tag{3.5}$$

In this process, V is increasing but the temperature is decreasing, hence

$$\frac{dT}{dV} \leq 0 \tag{3.6}$$

Using Eq. (3.5)

$$\frac{-2aV + b}{R} \leq 0 \tag{3.7}$$

$$\implies V \geq \frac{b}{2a} \tag{3.8}$$

This is the lower bound on the volume. This means if we want work done to be maximum

$$V_{\text{min}} = V_A = \frac{b}{2a} \tag{3.9}$$

Using the first law of thermodynamics $dQ = dU + PdV$,

$$dQ = \frac{R}{\gamma - 1} dT + PdV \tag{3.10}$$

where we use $dU = C_V dT$ and Eq.(3.2), Eq. (3.5), and $\gamma = 5/3$ in the above equation yields

$$dQ = \frac{3R(-2aV + b)dV}{2R} + (-aV + b)dV \tag{3.11}$$

$$\frac{dQ}{dV} = \left(-4aV + \frac{5b}{2}\right) \tag{3.12}$$

In the process, heat is taken and volume is also increasing. hence

$$\frac{dQ}{dV} \geq 0 \tag{3.13}$$

$$-4aV + \frac{5b}{2} \geq 0 \tag{3.14}$$

$$\implies V \leq \frac{5b}{8a} \tag{3.15}$$

This is the upper bound on the volume. This means if we want work done to be maximum

$$V_{\text{max}} = V_{\text{B}} = \frac{5b}{8a} \tag{3.16}$$

To get maximum work, The gas must expand from V_A to V_B

$$W_{\text{max}} = \int_{V_A}^{V_B} PdV = \int_{V_{\text{A}}}^{V_{\text{B}}} (-aV + b)dV \tag{3.17}$$

$$= \left[\frac{-aV^2}{2} + bV\right]_{b/2a}^{5b/8a} \tag{3.18}$$

Substituting the limits, we get,

$$W_{\text{max}} = \frac{7}{128}\frac{b^2}{a} \tag{3.19}$$

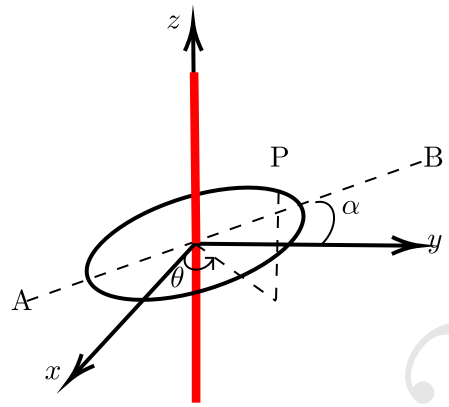
Solving $(-aV_A + b)V_A = 500R$, we get $\frac{b^2}{a} = 500 \times 4R$. Substituting this in the above equation, we get,

$$W_{\text{max}} \approx 909\,\text{J} \tag{3.20}$$

4. **Electrostatic TikTok**

Consider a fixed infinite vertical thin rod (shown by the red color in the figure below) of linear charge density λ along the z-axis at the origin (see figure below). A uniformly charged ring of total charge Q, mass M, and radius a is placed with its center at the origin in the x-y plane. Point P is an arbitrary point on the ring. The projection of point P on x-y plane makes an angle θ with respect to the x-axis in the anticlockwise direction as seen from the top.

The ring is now given an initial angular velocity ω_0 about the x-axis. We define the angle α which the plane of the ring makes with the x-y plane. This is illustrated by drawing line segment AB in the plane of the ring. Initially $\alpha = 0$. Ignore gravity.



You may find the following differentiation useful

$$D = \frac{d}{d\theta}[\tan^{-1}(q\tan\theta)] = \frac{1}{1+(q\tan\theta)^2}[q(\sec^2\theta)] \tag{4.1}$$

(a) **[1 marks]** State an expression for the electric field ($\vec{E}_0$) due to the infinite rod at a point on the ring when α=0 in terms of x, y and θ, and related quantities.

Solution: The electric field on the ring due to the infinite rod is given by

$$\vec{E}_0 = \frac{\lambda(x\hat{x}+y\hat{y})}{2\pi\epsilon_0(x^2+y^2)} \tag{4.2}$$

Since the rod is infinite, the electric field will not depend on z.

$$\vec{E}_0 = \frac{\lambda(a\cos\theta\hat{x}+a\sin\theta\hat{y})}{2\pi\epsilon_0(a^2\cos^2\theta+a^2\sin^2\theta)} \tag{4.3}$$

The above expression simplifies to

$$\vec{E}_0 = \frac{\lambda(\cos\theta\hat{x}+\sin\theta\hat{y})}{2\pi\epsilon_0 a} \tag{4.4}$$

(b) **[2 marks]** At some instant the ring makes an angle α. Derive an expression for the electric field $\vec{E}$ due to the infinite rod at a point on the ring in terms of θ, and α.

Solution: The new coordinates of the ring are $x = a\cos\theta$, $y = a\sin\theta\cos\alpha$, $z = a\sin\theta\sin\alpha$

$$\begin{aligned}\vec{E} &= \frac{\lambda(x\hat{x}+y\hat{y})}{2\pi\epsilon_0(x^2+y^2)} && (4.5)\\ &= \frac{\lambda(a\cos\theta\hat{x}+a\sin\theta\cos\alpha\hat{y})}{2\pi\epsilon_0(a^2\cos^2\theta+a^2\sin^2\theta\cos^2\alpha)} && (4.6)\\ &= \frac{\lambda(\cos\theta\hat{x}+\sin\theta\cos\alpha\hat{y})}{2\pi\epsilon_0 a(\cos^2\theta+\sin^2\theta\cos^2\alpha)} && (4.7)\end{aligned}$$

(c) **[1 marks]** Find the net force $\vec{F}$ acting on the ring.

Solution: Charge dQ in elementary length $ds = ad\theta$ is $\frac{Qd\theta}{2\pi}$. Force on small element ds is

$$dF = \vec{E}dQ = \frac{\lambda}{2\pi\epsilon a}\left[\frac{\cos\theta\hat{x} + \sin\theta\cos\alpha\hat{y}}{\cos^2\theta + \sin^2\theta\cos^2\alpha}\right]\frac{Qd\theta}{2\pi} \tag{4.8}$$

$$F = \int_{-\pi}^{\pi}\frac{\lambda}{2\pi\epsilon a}\left[\frac{\cos\theta\hat{x} + \sin\theta\cos\alpha\hat{y}}{\cos^2\theta + \sin^2\theta\cos^2\alpha}\right]\frac{Qd\theta}{2\pi} \tag{4.9}$$

$$= C\int_{-\pi}^{\pi}\frac{\cos\theta\hat{x} + \sin\theta\cos\alpha\hat{y}}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta \tag{4.10}$$

$$\tag{4.11}$$

where $C = \lambda Q/4\pi^2\epsilon_o a$. Consider

$$F_x = C\int_{-\pi}^{\pi}\frac{\cos\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta \tag{4.12}$$

since integrand is an even function

$$F_x = 2C\int_{0}^{\pi}\frac{\cos\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta \tag{4.13}$$

using $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$ in above equation, we get

$$= 2C\left[\int_0^{\pi/2}\frac{\cos\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta + \int_0^{\pi/2}\frac{\cos(\pi-\theta)}{\cos^2(\pi-\theta) + \sin^2(\pi-\theta)\cos^2(\alpha)}d\theta\right] \tag{4.14}$$

$$= 2C\left[\int_0^{\pi/2}\frac{\cos\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta - \int_0^{\pi/2}\frac{\cos\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta\right] \tag{4.15}$$

$$F_x = 0 \tag{4.16}$$

Similarly

$$F_y = C\int_{-\pi}^{\pi}\frac{\sin\theta\cos\alpha}{\cos^2\theta + \sin^2\theta\cos^2\alpha}d\theta \tag{4.17}$$

Here integrand is an odd function, hence

$$F_y = 0 \tag{4.18}$$

The total force acting on the ring is zero. Answers based on symmetric arguments will be also given credit.

(d) **[5 marks]** Find the net torque $\vec{\tau}$ acting on the ring in terms of α and the constants only. Qualitatively plot torque as a function of α.

Solution:

$$d\tau = \vec{r}\times d\vec{F}(\theta) \tag{4.19}$$

Consider

$$d\tau_z = xdF_y - ydF_x \tag{4.20}$$
$$= 0 \tag{4.21}$$
$$\implies \tau_z = 0 \tag{4.22}$$
$$\tau_y = \int_{-\pi}^{\pi} zdF_x = Ca\cos\alpha \int_{-\pi}^{\pi} \frac{\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha} d\theta \tag{4.23}$$

The integrand is an odd function

$$\implies \tau_y = 0 \tag{4.24}$$

$$\tau_x = \int_{-\pi}^{\pi} -zdF_y = -Ca\sin\alpha\cos\alpha \int_{-\pi}^{\pi} \frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha} d\theta \tag{4.25}$$

The integrand is an even function, hence

$$\tau_x = -2Ca\sin\alpha\cos\alpha \int_{0}^{\pi} \frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha} d\theta \tag{4.26}$$

using $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

$$\tau_x = -4Ca\sin\alpha\cos\alpha \int_{0}^{\pi/2} \frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha} d\theta \tag{4.27}$$

Consider

$$I = \int_{0}^{\pi/2} \frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta\cos^2\alpha} d\theta \tag{4.28}$$

Simplifying the above equation, we get

$$I = \int_{0}^{\pi/2} \frac{\tan^2\theta}{1 + \tan^2\theta\cos^2\alpha} d\theta \tag{4.29}$$

Substituting $\cos\alpha = u$, in above equation, we get

$$I = \int_{0}^{\pi/2} \frac{\tan^2\theta}{1 + u^2\tan^2\theta} d\theta \tag{4.30}$$

Substituting I in Eq.(4.27), we get

$$\tau_x = -4Ca\sin\alpha\cos\alpha \int_{0}^{\pi/2} \frac{\tan^2\theta}{1 + u^2\tan^2\theta} d\theta \tag{4.31}$$

It is given that

$$D = \frac{d}{d\theta}(\tan^{-1}(u\tan\theta)) = \frac{u\sec^2\theta}{1 + u^2\tan^2\theta} \tag{4.32}$$
$$= \frac{u(1+\tan^2\theta)}{1 + u^2\tan^2\theta} \tag{4.33}$$
$$D = \frac{u}{1 + u^2\tan^2\theta} + \frac{u\tan^2\theta}{1 + u^2\tan^2\theta} + u - u\frac{1 + u^2\tan^2\theta}{1 + u^2\tan^2\theta} \tag{4.34}$$

Solving above equation, we get

$$\frac{\tan^2\theta}{1 + u^2\tan^2\theta} = \frac{D}{u - u^3} - \frac{1}{1 - u^2} \tag{4.35}$$

Hence Substituting above equation in Eq.(4.31) , we get

$$\tau_x = -4Ca\sin\alpha u\int_0^{\pi/2}\left[\frac{D}{u-u^3} - \frac{1}{1-u^2}\right]d\theta \tag{4.36}$$

Substituting value of D, we get

$$\tau_x = -4Ca\sin\alpha u\int_0^{\pi/2}\left[\frac{\frac{d}{d\theta}(\tan^{-1}(u\tan\theta))}{u-u^3} - \frac{1}{1-u^2}\right]d\theta \tag{4.37}$$

$$\tau_x = -4Ca\sin\alpha u\left[\frac{\tan^{-1}(u\tan\theta)}{u-u^3} - \frac{\theta}{1-u^2}\right]_0^{\pi/2} \tag{4.38}$$

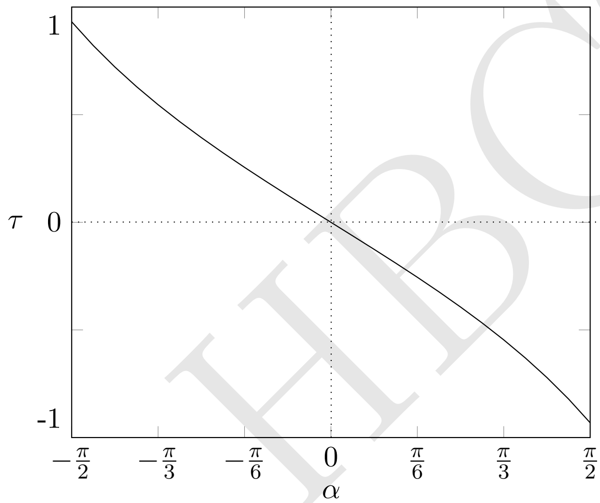
$$\tag{4.39}$$

Applying limits and solving further, we get

$$\tau_x = -\frac{\lambda Q}{2\pi\epsilon_0}\frac{\sin\alpha}{1+\cos\alpha} \tag{4.40}$$

$$\tau_x = -\frac{\lambda Q}{2\pi\epsilon_0}\tan(\alpha/2) \tag{4.41}$$

Working of τ_y, τ_z is not required.



(e) **[2 marks]** Let the ring is in equilibrium with respect to $\alpha = 0$. Derive an expression for the time period T of small oscillations of the ring in terms of λ, and Q. Take $\lambda = 0.1\mu\,\text{C/m}$, $Q = 2.0\,\mu\text{C}$, $M = 50.0\text{g}$, radius $a = 5.0\,\text{cm}$, and $\omega_0 = 1.0\,\text{rad/s}$. Calculate T.

Solution: Under small angle approximation of α, τ_x becomes

$$\tau_x = \frac{-\lambda Q\alpha}{4\pi\epsilon_0} \tag{4.42}$$

$$I\frac{d^2\alpha}{dt^2} = \frac{-\lambda Q\alpha}{4\pi\epsilon_0} \tag{4.43}$$

$$\frac{Ma^2}{2}\frac{d^2\alpha}{dt^2} = \frac{-\lambda Q\alpha}{4\pi\epsilon_0} \tag{4.44}$$

$$\frac{d^2\alpha}{dt^2} = -\frac{2\lambda Q}{4Ma^2\pi\epsilon_0}\alpha \tag{4.45}$$

This is a differential equation of SHM, hence

$$T^2 = \frac{4\pi^2}{\frac{2\lambda Q}{4Ma^2\pi\epsilon_0}} \tag{4.46}$$

$$\implies T = 2\pi a\sqrt{\frac{2M\pi\epsilon_0}{Q\lambda}} \tag{4.47}$$

$$T = 1.17\mathrm{s} \tag{4.48}$$

This can be used as an electrostatic clock!

(f) **[2.5 marks]** Find an expression for the potential energy U of the ring in terms of α. Qualitatively plot U as a function of α. Take the zero of potential energy to be at $\alpha = 0$.

Solution: We know that $\tau = -\frac{dU}{d\alpha}$

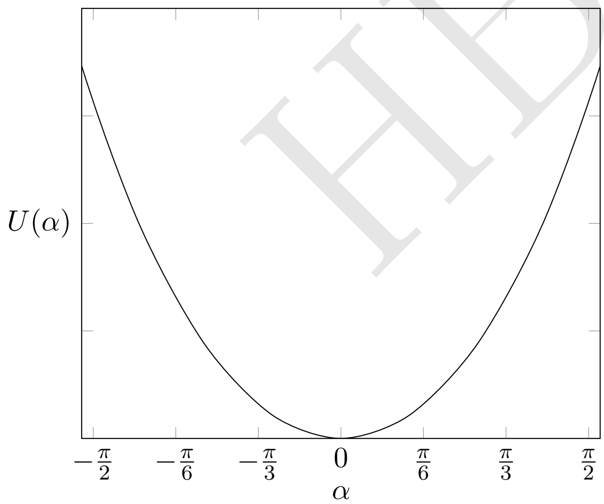
$$\implies U = -\int \tau d\alpha \tag{4.49}$$

$$= \frac{\lambda Q}{2\pi\epsilon_0}\int \tan(\alpha/2)d\alpha \tag{4.50}$$

$$U = -\frac{\lambda Q}{2\pi\epsilon_0}2\log(\cos(\alpha/2)) + c \tag{4.51}$$

where c is the constant of integration. At $\alpha = 0$, $U = 0$, which implies that $c = 0$.

$$\implies U(\alpha) = -\frac{\lambda Q}{\pi\epsilon_0}\log(\cos(\alpha/2)) \tag{4.52}$$



(g) **[2.5 marks]** Obtain the expression of maximum value of α ($\alpha_{\max}$) in terms of ω_0. Calculate $\alpha_{\max}$.

Solution: Consider

$$I\frac{d^2\alpha}{dt^2} = \tau_x \tag{4.53}$$

$$\frac{Ma^2}{2}\frac{d^2\alpha}{dt^2} = \frac{-\lambda Q}{2\pi\epsilon_0}\tan(\alpha/2) \tag{4.54}$$

Multiplying both sides by $\frac{d\alpha}{dt}$

$$\frac{Ma^2}{2}\frac{d^2\alpha}{dt^2}\frac{d\alpha}{dt} = \frac{-\lambda Q}{2\pi\epsilon_0}\tan\left(\alpha/2\right)\frac{d\alpha}{dt} \tag{4.55}$$

Integrating on both sides

$$\frac{Ma^2}{4}\left(\frac{d\alpha}{dt}\right)^2 = \frac{\lambda Q}{\pi\epsilon_0}\log(\cos\left(\alpha/2\right)) + c' \tag{4.56}$$

$$\left(\frac{d\alpha}{dt}\right)^2 = \frac{32\pi^2}{T^2}\log\left(\cos\left(\alpha/2\right)\right) + c' \tag{4.57}$$

where c' is the constant of integration. At $t = 0$, $\frac{d\alpha}{dt} = \omega_0$, which implies that $c = \omega_0^2$.

$$\left(\frac{d\alpha}{dt}\right)^2 = \frac{32\pi^2}{T^2}\log\left(\cos\left(\alpha/2\right)\right) + \omega_0^2 \tag{4.58}$$

$$\frac{d\alpha}{dt} = \sqrt{\frac{32\pi^2}{T^2}\log\left(\cos\left(\alpha/2\right)\right) + \omega_0^2} \tag{4.59}$$

For $\alpha = \alpha_{\max}$, $\frac{d\alpha}{at} = 0$, hence solving above equation, we get

$$\alpha_{\max} = 2\left[\cos^{-1}\left(\exp\left(-\frac{\omega_0^2 T^2}{32\pi^2}\right)\right)\right] \tag{4.60}$$

$$\alpha_{\max} = 10.66^\circ \tag{4.61}$$

5. **If Prof. Snell had a smartphone**

A typical smartphone screen is made up of mainly two components: a sheet of touch-sensitive glass (where you move your finger to operate the phone) of thickness t at the top and a LCD screen below it consisting of a regular array of "RGB elements" that emit light. These elements have a separation of d between them. There is a thin air gap of depth h between the touch-sensitive glass and the LCD screen (see Fig. (1)for a cross sectional view). We estimate the value of h from the following experiment.

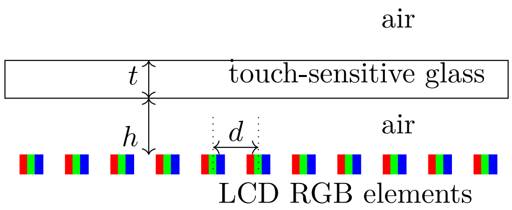


Figure 1

We use two smartphones (S-I and S-II) in this exercise – S-I is the target instrument in which we want to estimate h, and S-II is the measuring instrument that can capture photos of the screen of S-I which we then analyse using a image-processing software.

A digital image captured by the camera of a smartphone (S-II here) consists of discrete picture elements called pixels. The image captured by S-II is processed through a software. A red color reference line is drawn on the image (see Fig. 3(a)). The software plots the "brightness value" at every point of the reference line as a function of the number of pixels from the left end of the line. Thus, pixel number is a marker for distance here. First, we need to calibrate distance in terms of pixel number.

The phone S-I is kept horizontal and the display is kept ON. A ruler is placed on its screen. S-II is fixed above S-I to capture images. The image of the screen captured is shown in Fig (2).

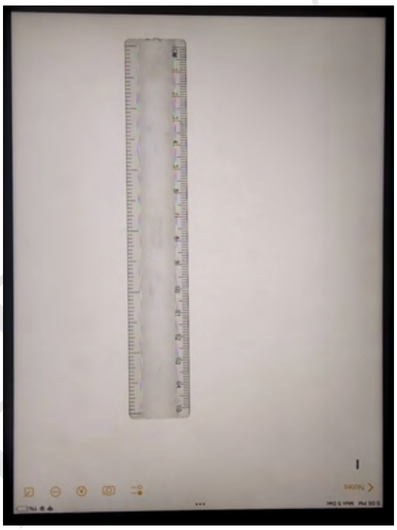
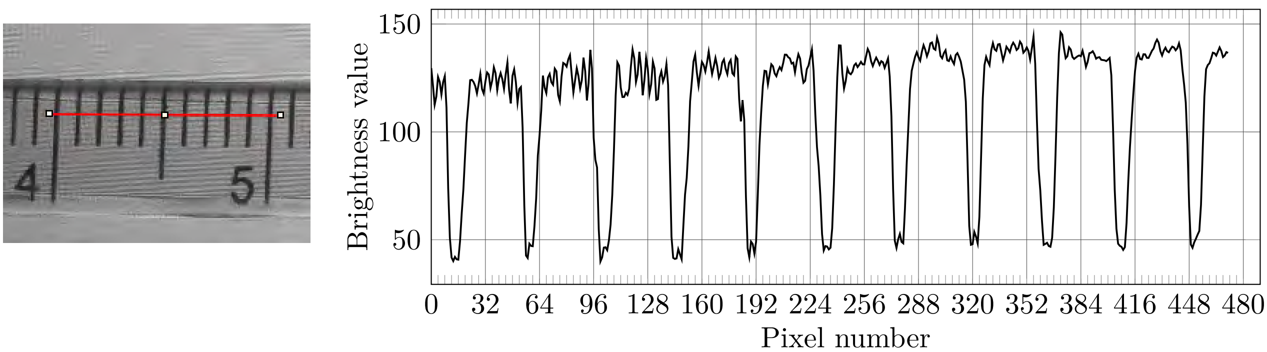


Figure 2

Figure 3(a) shows a part of the image of the ruler and its brightness value profile along the red reference line in Fig. 3(b).



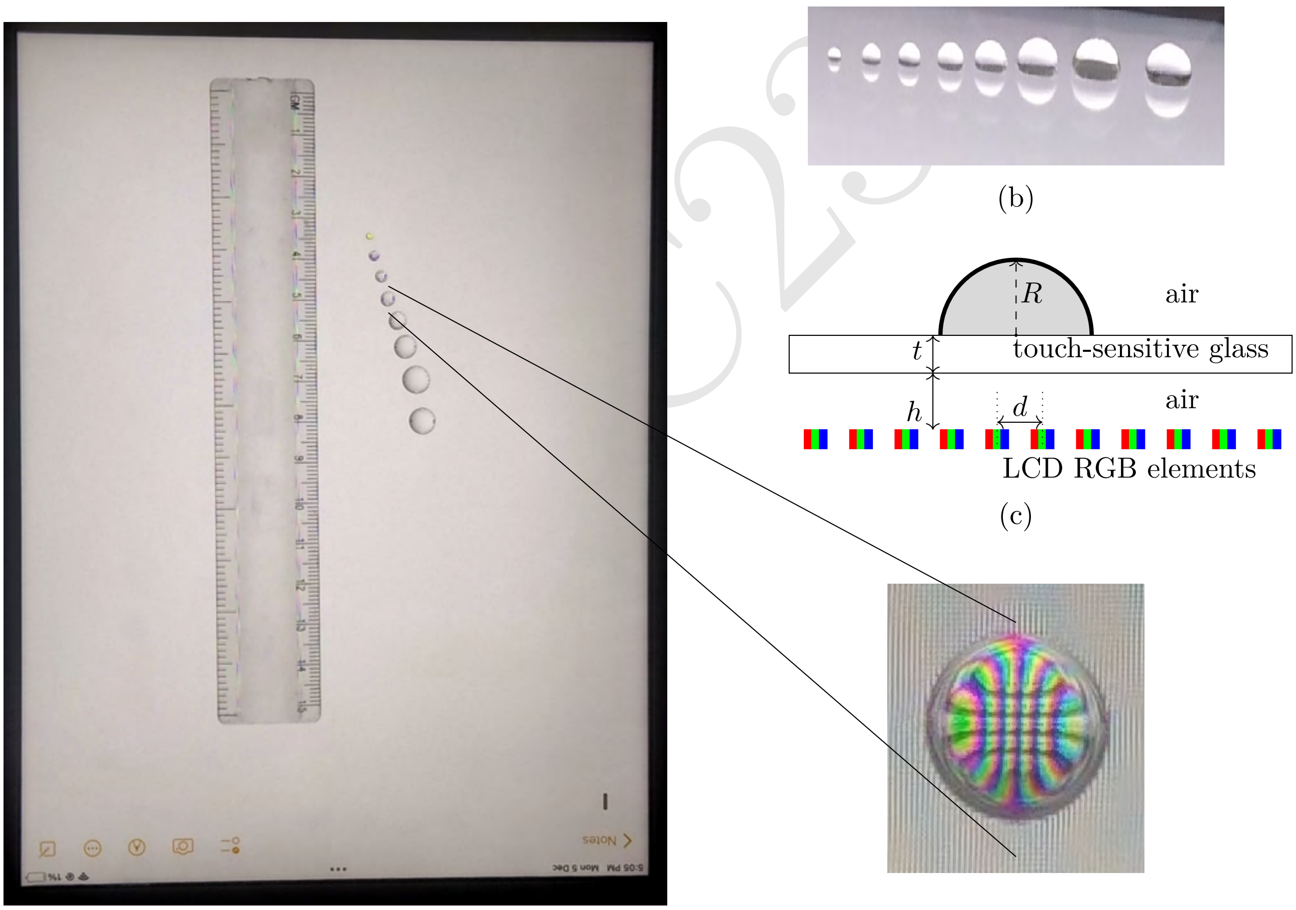
(a) (b)

Figure 3

(a) **[2 marks]** State the number of pixels used by the camera of S-II to capture one centimeter of the screen of S-I.

> **Solution:**
> Wherever there is a black color comes into the picture, the brightness value profile will show a dip. There is a dip at the pixel number 12 that refers to the 4 cm marker of the ruler. Similarly, the brightness value dip at the pixel number 452 is for the 5 cm marker of the scale. Hence the number of pixels present in 1cm of the image is 440. We denote the value $\theta = 1/440$ to be the scaling factor to convert the measurements obtained in pixels to the centimeter scale.
>
> Accepted answer range : $\dfrac{1}{440} to \dfrac{1}{436}$ pixels.

(b) **[5 marks]** We keep the setup the same as the last part. Next, a few small water drops are placed on the glass screen of S-I beside the ruler (see Figs. 4(a) and 4(b) for a top and side view, respectively). We model every drop as a hemispherical lens of radius R that magnifies the array of RGB elements of the LCD screen of S-I (see Fig. 4(c); the figure is not to scale).



(a) (d)

Figure 4

Figure 4(d) shows the magnified image of the array of the RGB elements of the screen as viewed from the top through one of the drops. This image is captured by S-II keeping the camera settings and distance same as in the previous part. The brightness value profiles of the images of the five chosen drops along the reference lines are shown in Fig. (5) on the next page.

Using the profile plots, write the radius of the water drop (R in mm) and the corresponding magnification (M) of the separation d between the array of RGB elements of S-I for each waterdrop lens. Use the table in the Summary Answer sheet to report your data. Describe the method you have used and the calculations in the Detailed Answer sheet.

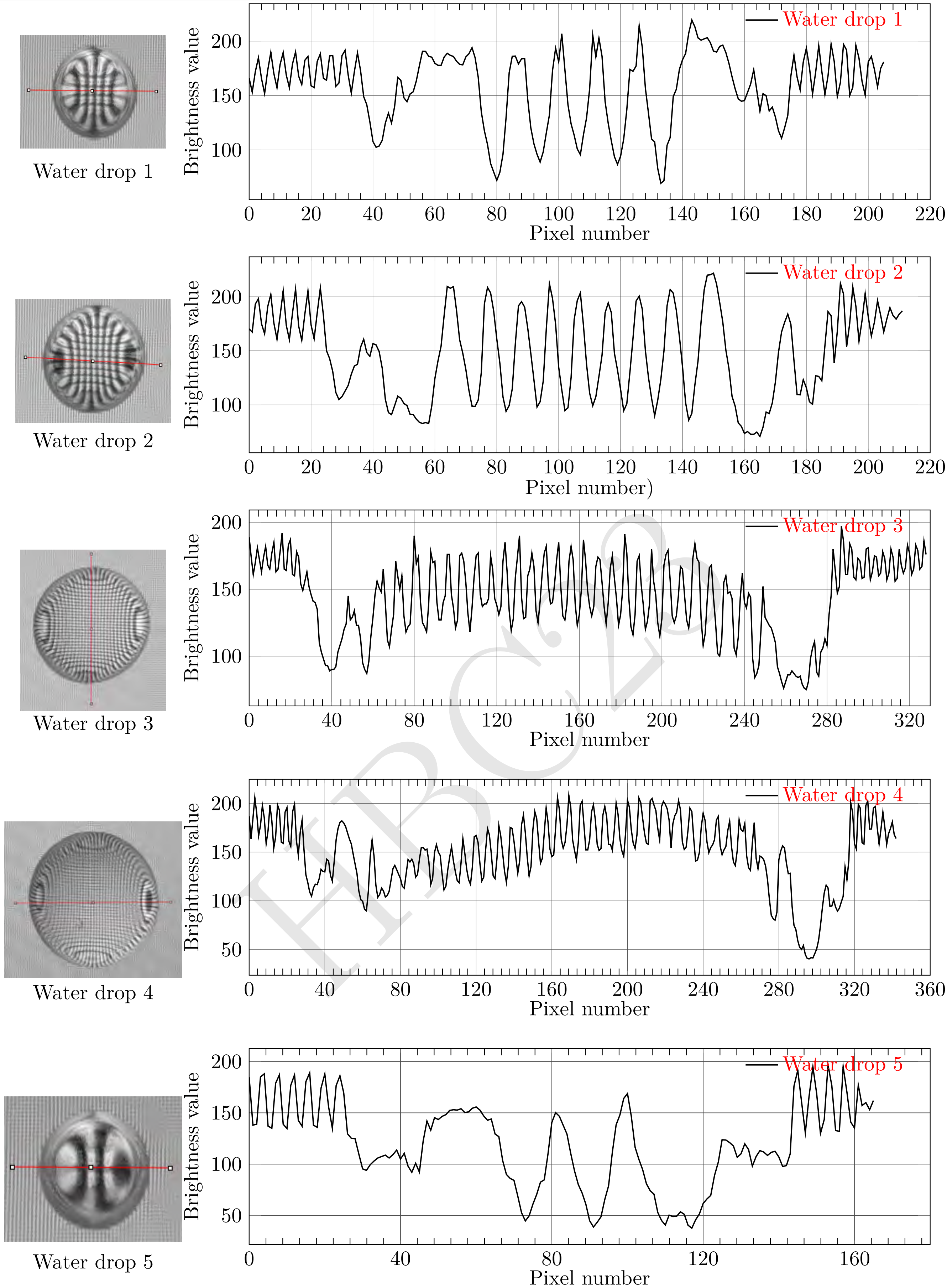


Figure 5: Question of part (b)

Solution: The red line is drawn beyond the waterdrops' diameters. In each brightness value profile, there are three distinct regions present. Reading from the left, a closely packed peaks,

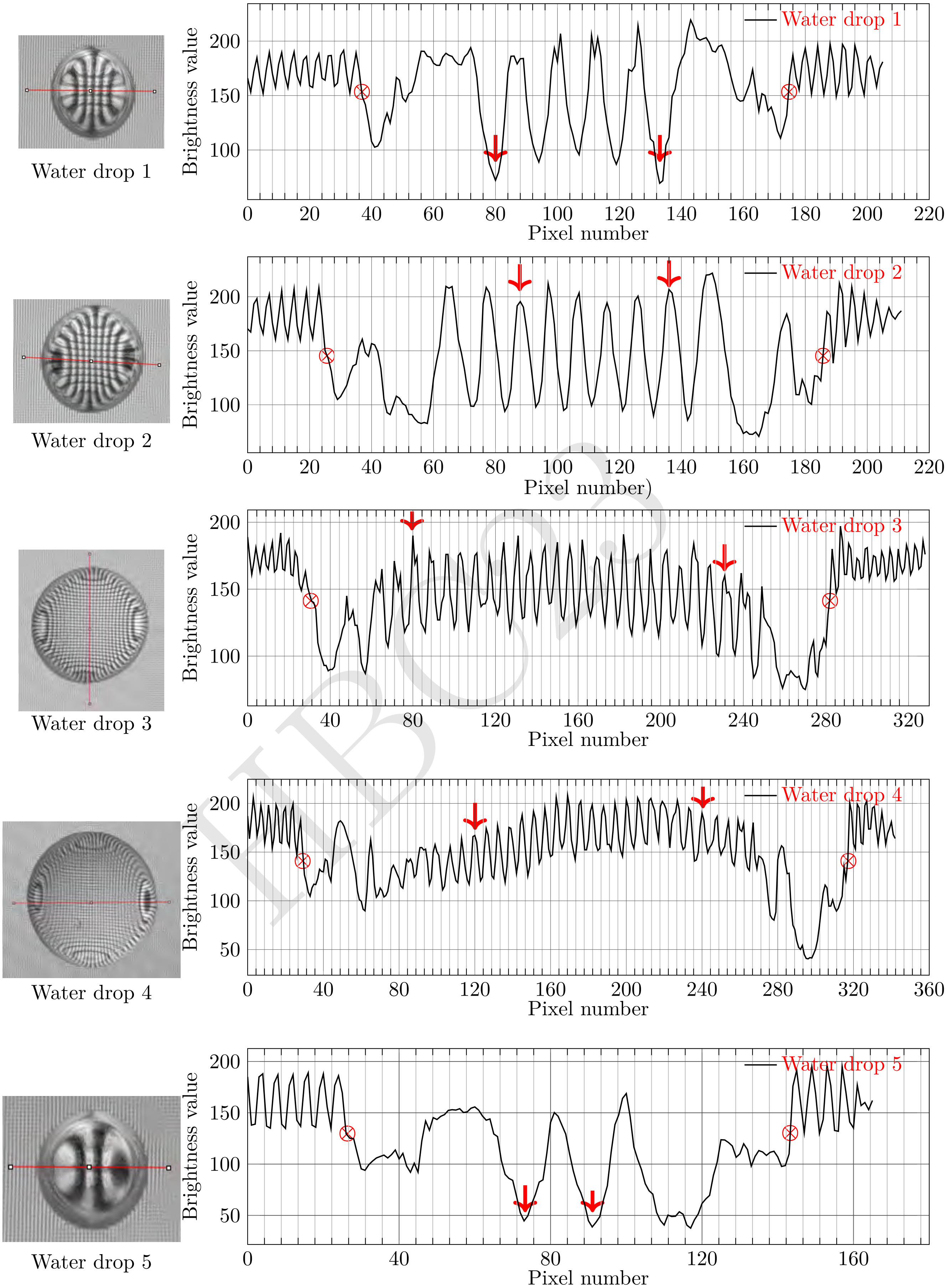


Figure 6: Solution of part (b)

then the central region where the peaks are dispersed and the right side to the central region is again a closely packed peaks. The right and left regions are the plots of the smartphone S-I's screen without the waterdrop lens. The distance d between each peak in these regions refers to the distance between the RGB elements of S-I. Note that d for each picture will be the same since all the images are extracted from one single image Fig. (4a).

The magnified distance D will be different for the drops, depending on the radius R. The central region is the magnified plot of the smartphone S-I screen seen through the waterdrop. The distance D between the two peaks in the central region is the magnified distance between S-I's RGB elements. The magnfication is D/d. For accuracy, we will count the n number of peaks for a distance and then divide the distance by n. The exact locations we have used on the plots to calculate D are indicated by a red color arrow $\downarrow$ (see Fig. (6)).

For each drop, we identify the pixel number which separates the waterdrop region. The distance along this region will be the diameter of the drop. Alternatively, you can also measure the length of the region with a physical ruler and then convert it into a pixel number. The boundary points of the waterdrop regions which we have used on the plots to calculate R are indicated by a red color symbol $\otimes$ (see Fig. (6)).

Every time we obtain the distance from the graph in terms of the pixel number, we multiply it by the scaling factor θ (obtained in part (a)) to convert it to the centimeter scale.

No.	Drop region		$^*R$ (cm)	Magnified distance			
				n	Distance		$^{**}D$(cm)
	Start pixel	End pixel			Start pixel	End pixel	
1	36	172	0.155	4	80	132	0.030
2	24	184	0.182	5	88	136	0.022
3	31.11	280	0.283	22	80	231.11	0.016
4	31.11	315.56	0.323	19	120	240	0.014
5	26.67	142.22	0.131	1	71.11	88.89	0.040

Here

$$^*R(\text{cm}) = \frac{\text{End pixel} - \text{Start pixel}}{2}\theta$$

$$^{**}D(\text{cm}) = \frac{\text{End pixel} - \text{Start pixel}}{n}\theta$$

and n is the number of peaks (or dips) counted.

The original distance d (unmagnified) between the RGB elements can be obtained by counting the dips in the left or right regions of any of the graphs. See the right side region of the water drop 1 graph, there are six peaks in 20 pixel numbers of the image, i.e. total of five RGB elements in 20 pixel numbers. Thus

$$d = \frac{20}{5}\theta \tag{5.1}$$

$$= \frac{1}{110}\text{cm} \tag{5.2}$$

Interesting fact for the readers: *RGB elements are nothing but the "pixels" inside S-I which you use to define the quality of a screen. When you refer to PPI (pixel per inch) of a phone, you are indicating the number of RGB elements in an inch of the screen display. We used ipad 8th generation as the S-I. Apple website gives PPI (pixel per inch) for the iPad to be 264* (https://support.apple.com/kb/SP822). *The value of d obtained gives the PPI value to be ∼279 PPI. Not a bad answer for an amateur setup!*

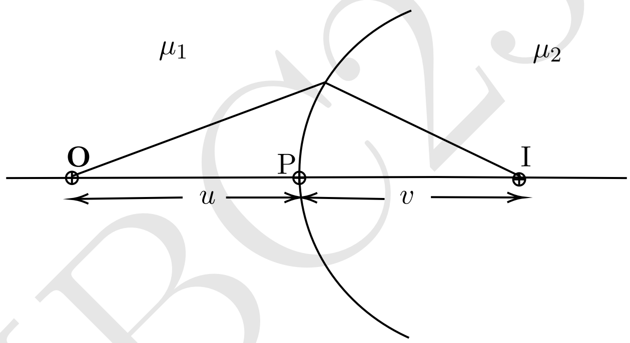
Data table for the Summary answer sheet:

Water drop	R (cm)	$M = D/d$
1	0.155	3.25
2	0.182	2.40
3	0.283	1.72
4	0.328	1.58
5	0.131	4.44

Final values within five percent of the official answers will be credited fully.

(c) **[9 marks]** For the given smartphone, $t = 0.50\,\text{mm}$, the refractive indices of the touch-sensitive glass, water drop, and the air to be $3/2, 4/3$, and 1 respectively. Using the data table of the previous part, plot a suitable linear graph to obtain the distance (h) of the RGB elements from the touch-sensitive glass. Use the table given in the summary answer sheet to enter the data used to plot the graph. Show your detailed theoretical calculation in the Detailed Answer sheet.

Solution: We use the standard results for the reflection formula from a spherical surface.

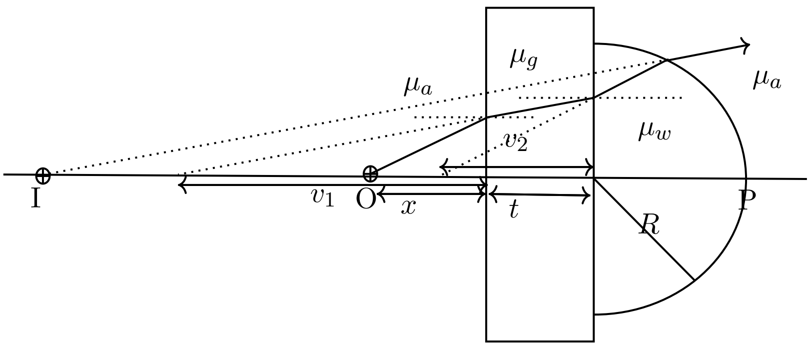


$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \tag{5.3}$$

$$\text{Magnification } M = \frac{I}{O} = \frac{\mu_1}{\mu_2}\frac{v}{u} \tag{5.4}$$

Here the symbols have their usual meanings. The sign will be adjusted accordingly.

There is refraction occurring at the three surfaces.



The first refraction is at the air-glass interface. Using $\mu_1 = \mu_a,\ \mu_2 = \mu_g,\ R = \infty$ in the Eq.(5.3)

$$\frac{\mu_g}{v_1} - \frac{\mu_a}{-x} = 0 \tag{5.5}$$

$$v_1 = -\mu_g x \tag{5.6}$$

The second refraction is at the glass-water interface. Now $u_2 = |v_1| + t$. This gives

$$\frac{\mu_w}{v_2} - \frac{\mu_g}{-u_2} = 0 \tag{5.7}$$

$$v_2 = -(\mu_g x + t)\frac{\mu_w}{\mu_g} \tag{5.8}$$

The third refraction is at the water-air interface. Now $u_3 = |v_2| + R$ gives

$$\frac{\mu_a}{v_3} - \frac{\mu_w}{-u_3} = \frac{\mu_a - \mu_w}{-R} \tag{5.9}$$

Magnification will only be from the third interface.

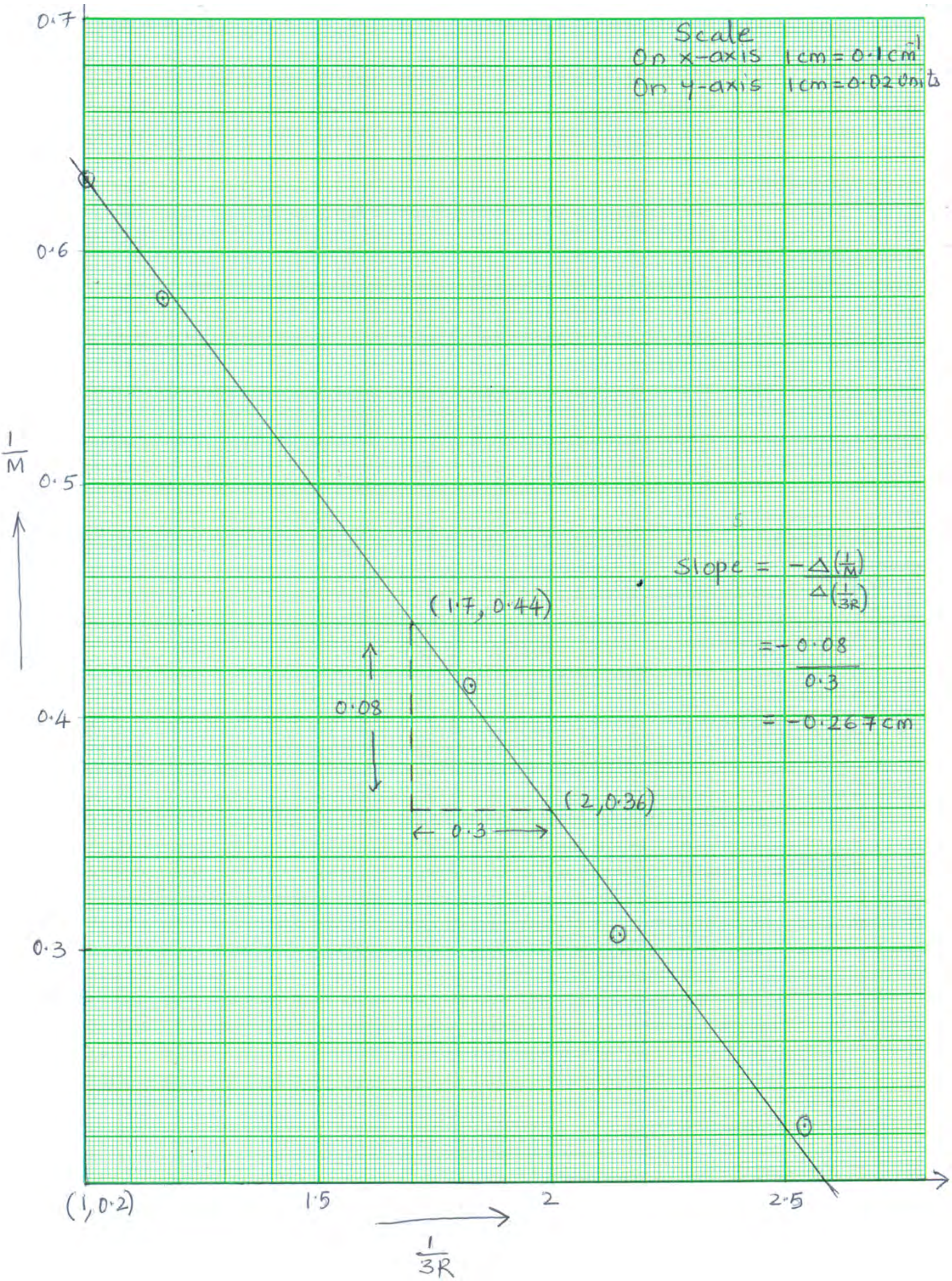
$$M = \frac{\mu_w}{\mu_a}\frac{v_3}{u_3} \tag{5.10}$$

Using $\mu_g = 3/2$, $\mu_w = 4/3$, and $\mu_a = 1$ in the Eqs. (5.9 and 5.10) yields

$$\frac{1}{M} = \frac{3}{4} - \frac{1}{3R}\left(x + \frac{2t}{3}\right) \tag{5.11}$$

A graph of $1/M$ vs $1/3R$ will be linear. The graph is plotted on the next page. For the obtained data set
Slope = 2.67mm which gives $x = 2.34$ mm.

Accepted answer range: (2.34±5%)mm.



**** END OF THE QUESTION PAPER ****

Indian National Physics Olympiad (INPhO)-2024

HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
Tata Institute of Fundamental Research
V. N. Purav Marg, Mankhurd, Mumbai, 400 088

Question Paper

Date: 04 February 2024
Time: **09:00-12:00 (3 hours)** Maximum Marks: **80**

Instructions **Roll No.:**

1. This booklet consists of 23 pages and total of 6 questions. Write roll number at the top wherever asked.

2. Booklet to write the answers is provided separately. Instructions to write the answers are on the Answer Booklet.

3. Non-programmable scientific calculators are allowed. Mobile phones **cannot** be used as calculators.

4. **Please submit the Answer Sheet at the end of the examination.** You may retain the Question Paper.

Table of Constants

Universal constant of Gravitation	G	6.67×10^{-11} N·m^2·kg^{-2}
Value of $1/4\pi\epsilon_0$		9.00×10^9 N·m^2·C^{-2}
Avogadro's number	N_A	6.022×10^{23} mol^{-1}
Acceleration due to gravity	g	9.81 m·s^{-2}
Universal Gas Constant	R	8.31 J· K^{-1}·mol^{-1}
	R	0.0821 l·atm·mol^{-1}·K^{-1}
Molar mass of Helium		4.003 g· mol^{-1}
Boltzmann constant	k_B	1.3806×10^{-23} J·K^{-1}
Earth's radius	R_E	6371km
Earth's mass	M_E	5.97×10^{24}kg
Moon's mass	M_M	7.35×10^{22}kg
Moon's radius	R_M	1737km

Question Number	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Maximum Marks	8	18	14	11	18	11	80

Please note that alternate/equivalent methods and different way of expressing final solutions may exist. A correct method will be suitably awarded.

1. **[8 marks] An electrifying experiment**
Professor Coulomb was investigating how the magnitude of the force ($|\vec{F}|$) between two charged spheres depends on the distance between their centres. He conducted four separate experiments by placing two identical conducting spheres 1 and 2, each of radius a, at different distances d from each other. The experiments are outlined in the table below. Here Q_1 and Q_2 are the charges on the spheres 1 and 2, respectively. The measurement results are presented in the graph.

Experiment no.	a (m)	Q_1	Q_2
1	0.10	$+Q$	$+Q$
2	0.10	$+Q$	$-Q$
3	0.05	$+Q$	$+Q$
4	0.05	$+Q$	$-Q$

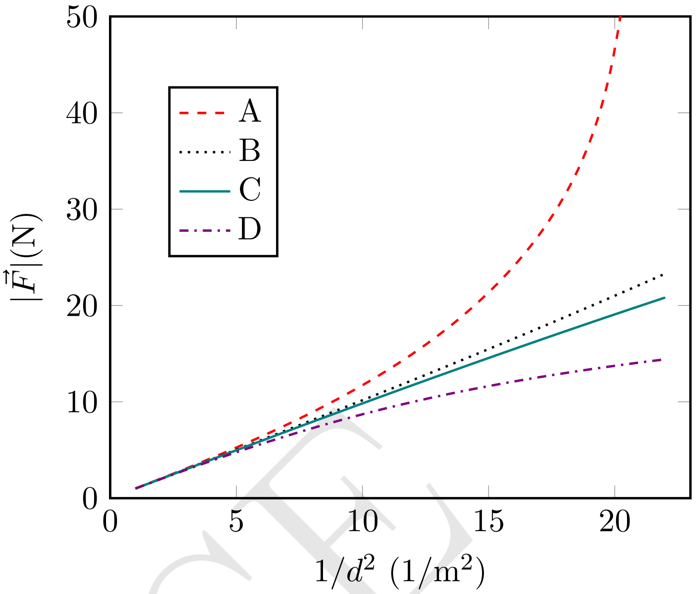
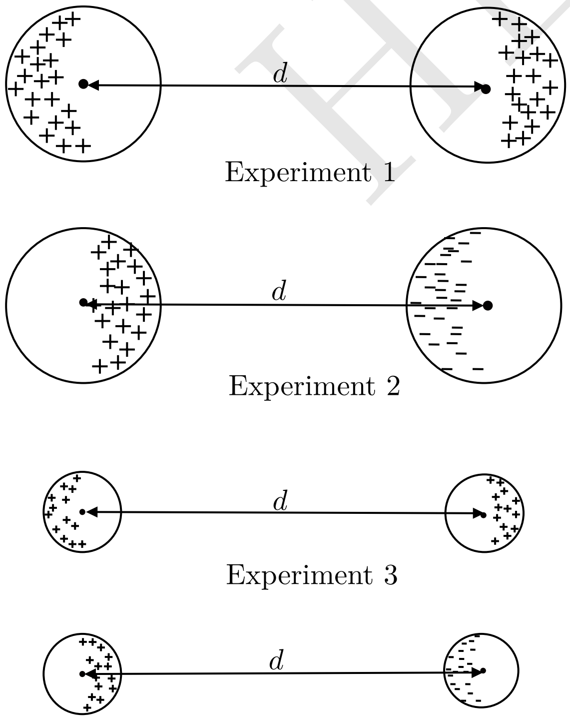


Figure out which measurement (A, B, C, D) belongs to which experiment (1, 2, 3, 4). Explain your answers in the detailed answersheet. You may draw diagrams, if necessary.

Solution: Here conducting spheres exhibit polarization when brought close, impacting the effective distance between charges. For two positively charged spheres at a distance d from their centers, polarization increases the effective distance, reducing the force. Similarly, for a positively and a negatively charged sphere, also at distance d from their centers, polarization decreases the effective distance and increases the force. The order of effective distances in the four experiments is $d_1^{\text{eff}} > d_3^{\text{eff}} > d_4^{\text{eff}} > d_2^{\text{eff}}$. This is depicted in the figure below.



Experiment 1

Experiment 2

Experiment 3

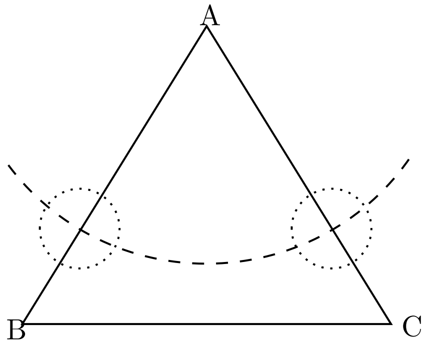
Experiment 4

The correct choices will be:
Experiment 1: D
Experiment 2: A
Experiment 3: C
Experiment 4: B

2. **A Potpourri of Prism Problems**

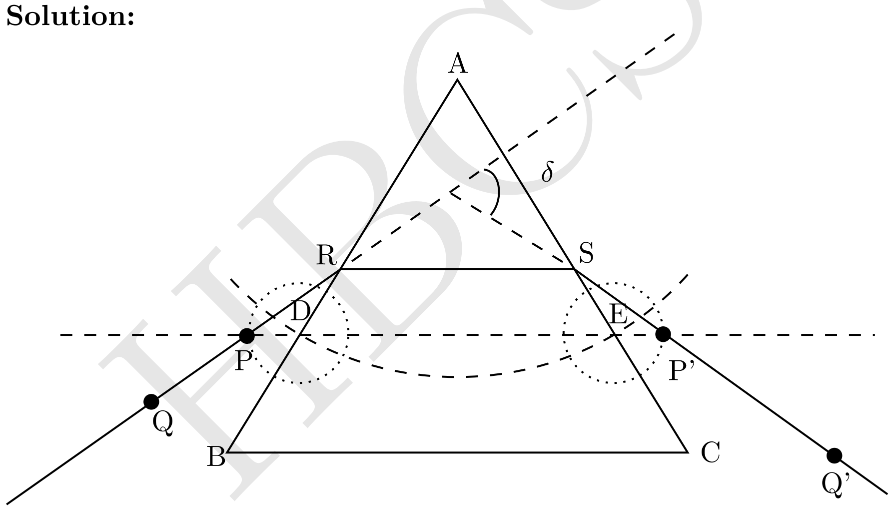
(a) **[7 marks]** In the most common method to determine the angle of minimum deviation by a prism, we record the angles of deviation (δ) for various angles of incidence (i) and then plot a graph. However, Professor Joseph proposed an ingenious idea to determine the angle of minimum deviation with just a single angle of incidence. Eager to share his breakthrough, he penned a letter to his friend, Professor Amal Nathan, outlining his method for an equilateral prism. According to Professor Joseph, the only tools required were four pins, a board, a marker pen or pencil, a scale, a protractor, and, of course, the prism. He even claimed that one may not need all the materials listed.

In his letter, Professor Joseph began to sketch a figure to illustrate the method. Here triangle ABC is the trace of the prism. The dashed curve is an arc of a circle centered at A. The dotted circles are centered on the points of intersection of the arc with the sides of the triangle. Unfortunately, he forgot to complete the figure. Can you describe the experimental method to determine the angle of minimum deviation using the unfinished figure of Professor Joseph? You must provide the following:



1. A complete ray diagram using the given figure. You may use the one already provided on the answersheet or draw a fresh one.
2. Outline of the essential experimental steps using some or all of the equipment mentioned above and nothing more. Use the detailed answer sheet for this.

Solution:

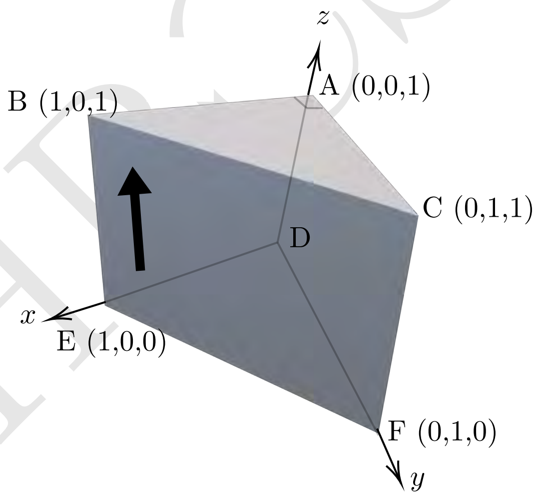


The big arc is drawn to draw a line parallel to the base BC, with A as the center. Smaller dotted circles are then drawn, centering on points D and E. The goal is to minimize the angle of deviation, which occurs when the refracted ray is parallel to the base. To observe and measure this deviation, the following steps are taken:

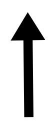
1. Draw a line passing through D and E. This is the dashed line parallel to the base BC.
2. Mark point P, where the circle centered at D intersects the line DE. Similarly, mark P'.
3. Mount pins on the points P and P'.
4. Mount 3rd pin at a position Q on the left side, such that pins at P, P', and Q appear collinear when observed from the right side refracting face of the prism.
5. In a similar manner looking from the left side, align the 4th pin labeled as Q' on the right side, such that pins at P, P', Q, and Q' appear collinear.

6. Observe the incident, refracted, and emergent rays formed by the pin's positions and the prism.
7. Extend the lines drawn to measure the angle of deviation.
8. Since PD=P'E, the refracted ray is parallel to the base, and the angle δ represents the minimum deviation.
9. Utilize a protractor to measure the angle.

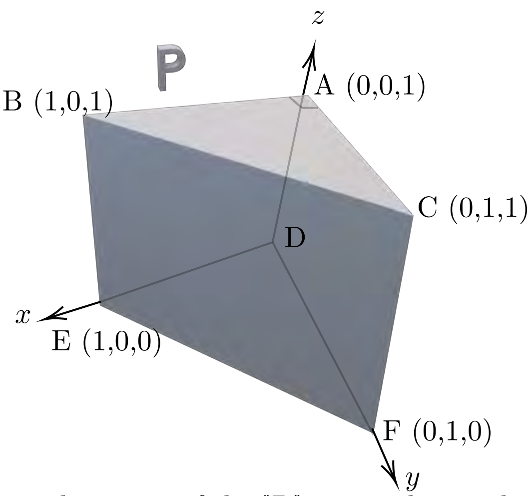
(b) **[1.5 marks]** Consider a right-angled isosceles prism as depicted below. The prism is placed on a table (x-y plane). The triangular faces are non-refracting surfaces. The refractive index of the prism is 1.50. The sides AB = AC = AD = 1 unit. The prism is positioned such that point D is at the origin, with the axes defined in the figure. An arrow-shaped object is pasted on the face BCFE of the prism as shown. Draw the image of the object as seen by an observer in front of the face BCFE.

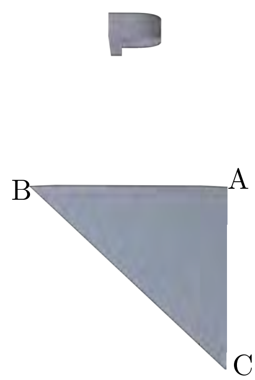


Solution: The formed image is virtual and located behind face ACDF when seen from face BCFE. The pasted arrow is symmetric about edge AD. It undergoes two total internal reflections, resulting in the image being identical to the object but laterally shifted.



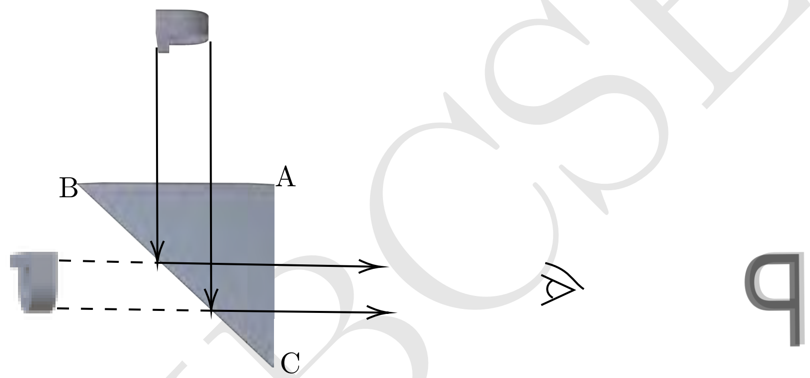
(c) **[1.5 marks]** Now an object, shaped like the letter "P" as illustrated in the left figure below, is held in front of the face ABED of the prism, placed on a table (x-y plane). The corresponding top view of this configuration is also presented in the right figure below.



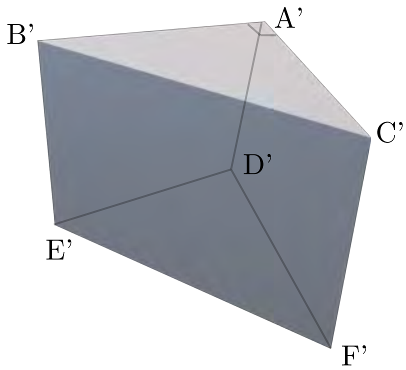


Draw the image of the "P" as seen by an observer in front the face ACFD. Additionally, draw a qualitative ray diagram illustrating the image formation.

Solution: The formed image is virtual and positioned behind face BCFE when observed facing face ACDE. It undergoes total internal reflection. Ray diagram and the resultant image is shown below.



(d) **[5 marks]** In this part, alongside the setup of part (c) with the prism ABCDEF and the object "P" positioned in front of its face ABED, we introduce another identical prism, A'B'C'D'E'F' (see below).



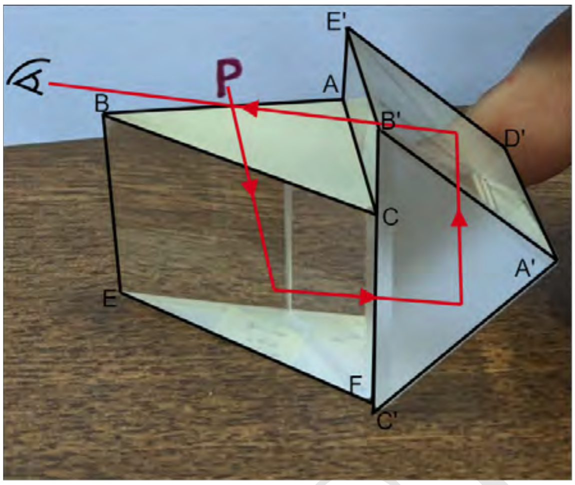
A specific experiment requires placing the prism A'B'C'D'E'F' in combination with the existing setup so that the following image (as shown below) of the object "P" can be obtained.

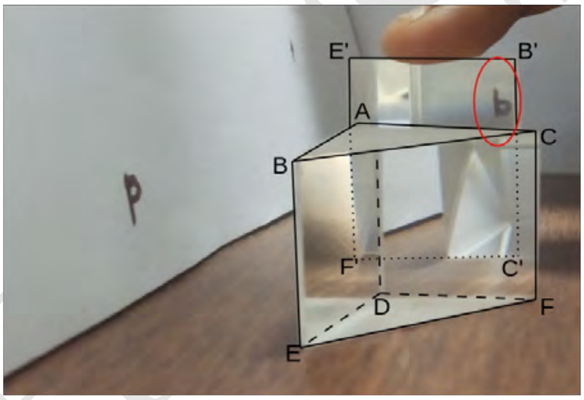
Without disturbing the prism ABCDEF, where and how would you hold the prism A'B'C'D'E'F' to achieve this resultant image?

Provide your answer in terms of the coordinates of the vertices of the prism A'B'C'D'E'F', taking vertex D of the first prism as the origin. Additionally, specify that to view such an image, the viewer should be facing which particular face of the prism.

Hint: The position of the second prism is such that any one of the parts (such as at least one of the vertices, edges, or faces) touches the table. The viewer should be positioned in a way that allows a clear line of sight to the area where the image is formed, ensuring an unobstructed view of the desired image.

Solution:





The light rays from object "P" initially pass through faces ABED of prism 1 at a normal angle. Subsequently, due to total internal reflection, the rays are reflected from face BCFE. The reflected light exits prism 1 through face ACFD and enters prism 2 through face E'B'C'F'. Inside prism 2, the light undergoes total internal reflection twice, occurring at faces A'C'F'D' and A'D'E'B'. The image formed is virtual and located behind the face C'F'E'B' face of the prism.

Face **C'F'E'B'** of the prism 2 can be kept in the plane **YZ**, touching the face **CADF** of the prism 1.

The position of the observer is in the plane **YZ and facing C'F'E'B' face of the second prism.**

Coordinates of the face **C'F'E'B'** are

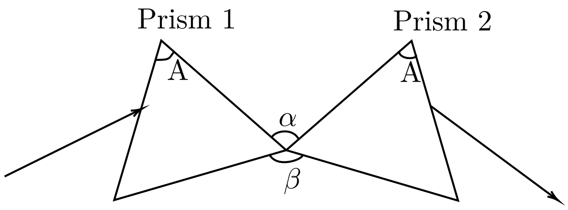
C': $(0, 1, 0)$

F': $(0, 0, 0)$

E': $(0, 0, \sqrt{2})$

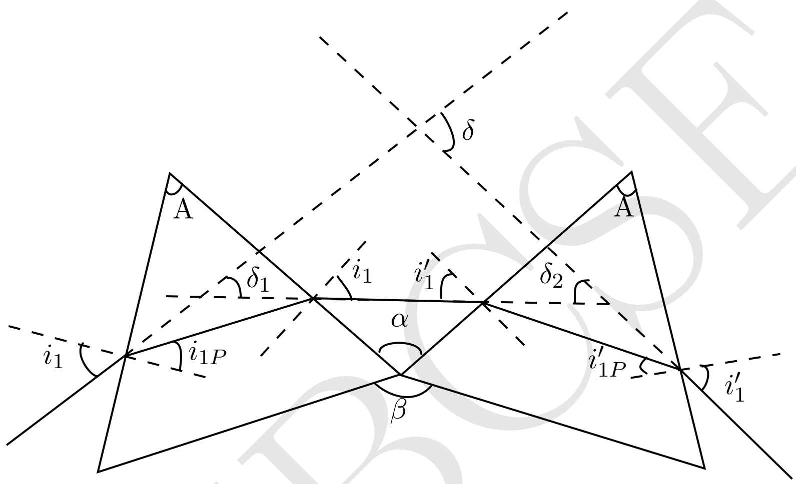
B': $(0, 1, \sqrt{2})$

(e) **[3 marks]** In a spectrograph, two equilateral prisms denoted as 1 and 2 with refractive indices $\mu_1 = 1.50$ and $\mu_2 = 1.68$, respectively, are placed one after another (see the figure below). The incident ray is shown on the left and the final emergent ray is shown on the right.



Find the angle (β) between the bases of the two prisms if each prism is individually adjusted for minimum deviation for the respective incident rays. Obtain the total deviation δ of the beam of light in this configuration. Express your answers in terms of μ_1, μ_2, A. Calculate β and δ in degrees.

Solution: Let angle of incidence be i_1 and for this ray, the angle of emergence from the second prism be i'_1. The angle of refraction at first prism be i_{1P}. The angle of incidence of light for second prism is i'_1, as it is adjusted for minimum deviation. Angle of refraction for ray going from air to 2nd prism be i'_{1P}.



$$\sin i_1 = \mu_1 \sin \frac{A}{2} \tag{2.1}$$

$$\sin i'_1 = \mu_2 \sin \frac{A}{2} \tag{2.2}$$

From geometry

$$\alpha = i_1 + i'_1 \tag{2.3}$$

$$\alpha = \sin^{-1}\left(\mu_1 \sin \frac{A}{2}\right) + \sin^{-1}\left(\mu_2 \sin \frac{A}{2}\right) \tag{2.4}$$

$$\beta = 240 - (i_1 + i'_1) \tag{2.5}$$

$$\beta = 240 - (\sin^{-1}\left(\mu_1 \sin \frac{A}{2}\right) + \sin^{-1}\left(\mu_2 \sin \frac{A}{2}\right)) \tag{2.6}$$

For $\mu_1 = 1.50$, $\mu_2 = 1.68$ and $A = 60°$, we get

$$\alpha = 105°44' \tag{2.7}$$

$$\beta = 134°16' \tag{2.8}$$

From figure and geometry If the angle of minimum deviation of the first prism is δ_1 and the second prism is δ_2., then

$$A + \delta_1 = 2i_1 \tag{2.9}$$

$$A + \delta_2 = 2i'_1 \tag{2.10}$$

The total deviation produced by two prisms is

$$\delta = \delta_1 + \delta_2 \tag{2.11}$$
$$\delta = 2(i_1 + i_1' - A) \tag{2.12}$$
$$\delta = 2(\sin^{-1}\left(\mu_1 \sin\frac{A}{2}\right) + \sin^{-1}\left(\mu_2 \sin\frac{A}{2}\right) - A) \tag{2.13}$$
$$= 91°28' \tag{2.14}$$

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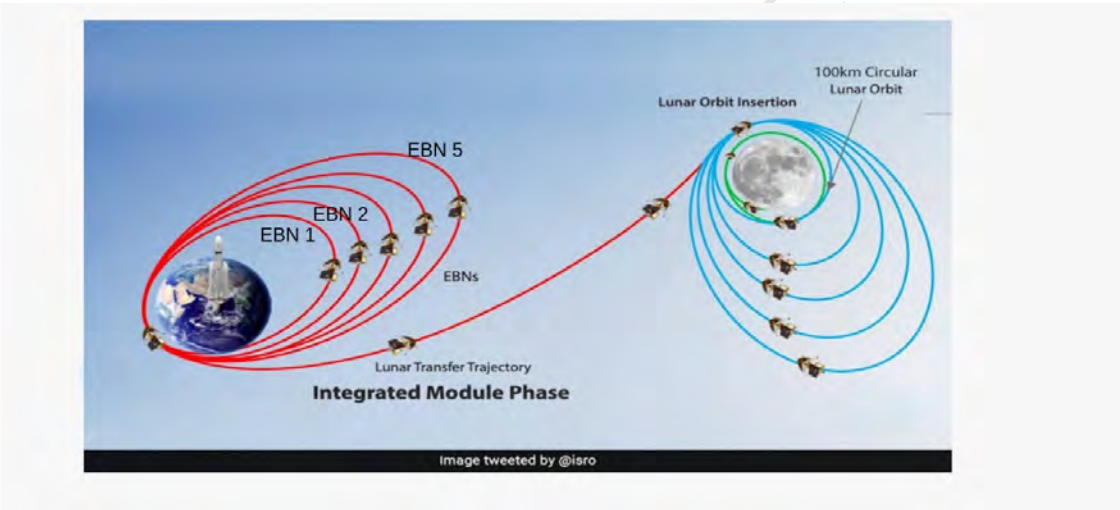
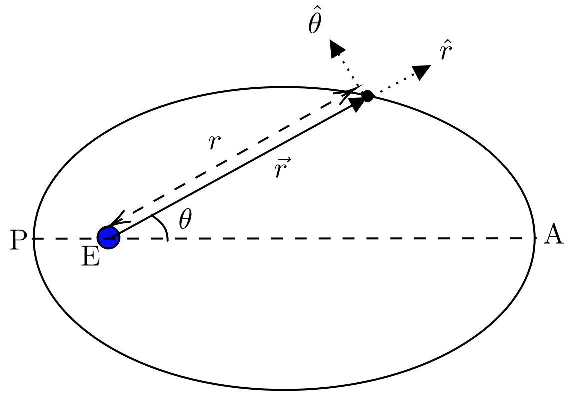
3. **Chandrayaan-3**

On July 14, 2023, India's lunar mission satellite, *Chandrayaan-3*, was successfully launched by the Indian Space Research Organization (ISRO). *Chandrayaan-3* (mass m =3900 kg) was taken to the Moon through a series of Earth Bound Manoeuvres (elliptical) orbits (EBNs) as depicted in the figure below. In this problem, we will explore the physics governing some part of its journey, employing a simplified model. For all parts of this problem except part (f), we consider *Chandrayaan-3* to be moving only under the influence of Earth's gravity (a central force).



(a) **[6 marks]** Upon launch, *Chandrayaan-3* entered an elliptical orbit around Earth, with Earth at one of the foci (E) as shown below. The points P and A are the perigee (nearest point from the Earth) and apogee (farthest point from the Earth), respectively. We introduce the polar coordinate system (r,θ), where $\vec{r}$ is the vector from the centre of the Earth (origin) to the satellite, and θ is the angle that $\vec{r}$ makes with the major axis (PA= $2a$). The directions of unit vectors $\hat{r}$ and $\hat{\theta}$ are shown in the figure.



The equation of the ellipse can be written in polar coordinates as

$$r = \frac{r_0}{(1 - e\cos\theta)}$$

where e is eccentricity of the orbit ($0 < e < 1$) and r_0 is called the latus rectum.
The velocity $\vec{v}$ of the satellite in polar coordinates can be written as

$$\vec{v} = v_r\hat{r} + v_t\hat{\theta} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

where $v_r = \dot{r}$ is the "radial" speed and $v_t = r\dot{\theta}$ is the "tangential" speed.
Make schematic plots of the speeds v_r and v_t as functions of θ over one full orbit. Mark any significant points in the plots in terms of a, e, and other variables.

Solution: It is given that

$$r = \frac{r_0}{(1 - e\cos\theta)} \tag{3.1}$$

$$\therefore v_r = \dot{r} = -\frac{r_0 e \sin\theta\dot{\theta}}{(1 - e\cos\theta)^2} \tag{3.2}$$

Since the force is central, angular momentum l is conserved. The conserved angular momentum is given by

$$l = mr^2\dot{\theta} \tag{3.3}$$

$$\implies \dot{\theta} = \frac{l}{mr^2} \tag{3.4}$$

Substituting above equation and expression of r in v_r, after simplifying, we get

$$v_r = -\frac{el\sin\theta}{mr_0} \tag{3.5}$$

We know that

$$v_t = \frac{l}{mr} \tag{3.6}$$

Substituting the value of r, we get

$$v_t = \frac{l(1 - e\cos\theta)}{mr_0} \tag{3.7}$$

From above equation, the value of v_t is maximum, when $\cos\theta$ is minimum. i.e. $\theta = -\pi$. The maximum value of v_t

$$v_t^{\max} = \frac{l(1+e)}{mr_0} \tag{3.8}$$

From Eq. (3.7), v_t is minimum, when $\cos\theta$ is maximum. i.e $\theta = 0$. The minimum value of v_t

$$v_t^{\min} = \frac{l(1-e)}{mr_0} \tag{3.9}$$

Since the velocities at P and A are purely tangential, and P is closer to earth as compared to A, hence the tangential speed is maximum at P. Let the velocity of the satellite in the given orbit at P be v_p and velocity at A be v_a. Conservation of energy gives:

$$\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = \frac{1}{2}mv_p^2 - \frac{GMm}{r_p} \tag{3.10}$$

Conservation of angular momentum gives

$$mv_a r_a = mv_p r_p \tag{3.11}$$

$$v_p = v_a\frac{r_a}{r_p} \tag{3.12}$$

Substituting the above equation into the energy conservation equation, we get

$$v_p = \sqrt{\frac{GM}{a}\frac{(1+e)}{(1-e)}} \tag{3.13}$$

Similarly

$$v_a = \sqrt{\frac{GM}{a}\frac{(1-e)}{(1+e)}} \tag{3.14}$$

Hence the conserved angular momentum can be written as

$$l = mv_a r_a = mv_a r_a \tag{3.15}$$

On simplification, we get

$$l = mr_o\sqrt{\frac{GM}{a(1-e^2)}} \tag{3.16}$$

v_r and v_t can also be written as

$$v_r = -e\sqrt{\frac{GM}{a(1-e^2)}}\sin\theta \tag{3.17}$$

$$v_t = \sqrt{\frac{GM}{a(1-e^2)}}(1-e\cos\theta) \tag{3.18}$$

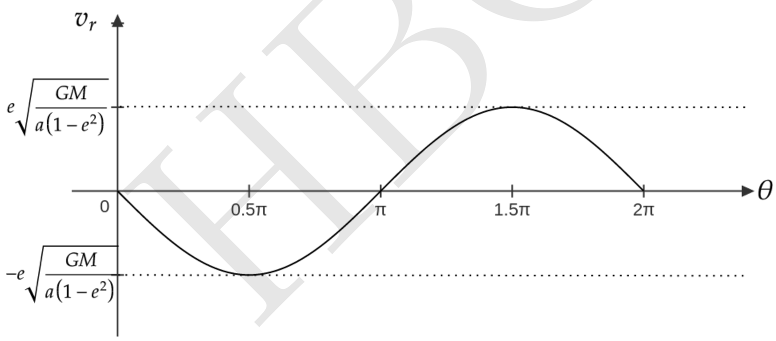
Similarly v_r^{max}, v_r^{min}, v_t^{max} and v_t^{min} can also be written as

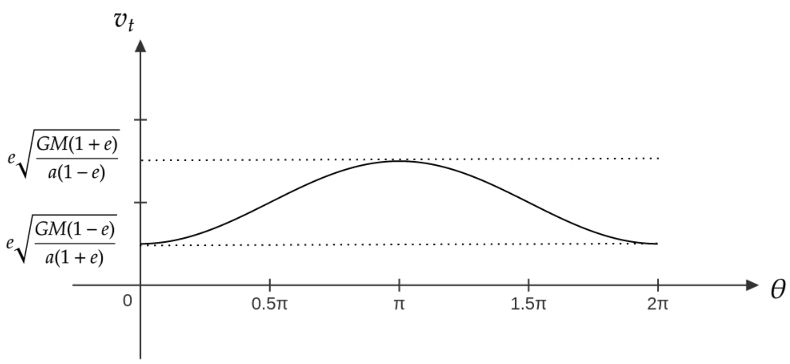
$$v_r^{max} = e\sqrt{\frac{GM}{a(1-e^2)}} \quad \text{at } \theta = 3\pi/2 \tag{3.19}$$

$$v_r^{min} = -e\sqrt{\frac{GM}{a(1-e^2)}} \quad \text{at } \theta = \pi/2 \tag{3.20}$$

$$v_t^{max} = \sqrt{\frac{GM}{a(1-e^2)}}(1+e) = \sqrt{\frac{GM(1+e)}{a(1-e)}} \quad \text{at } \theta = \pi \tag{3.21}$$

$$v_t^{min} = \sqrt{\frac{GM}{a(1-e^2)}}(1-e) = \sqrt{\frac{GM(1-e)}{a(1+e)}} \quad \text{at } \theta = 0 \text{ and } 2\pi \tag{3.22}$$





(b) **[1.5 marks]** Obtain an expression for the total energy (E) of the orbiting satellite in terms of a and other constants.

Solution: Let

$$\frac{v_a}{r_p} = \frac{v_p}{r_a} = C \tag{3.23}$$

Putting Eq. (3.23) in Eq. (3.10), we get

$$\frac{1}{2}mC^2r_p^2 - \frac{GMm}{r_a} = \frac{1}{2}mC^2r_a^2 - \frac{GMm}{r_p} \tag{3.24}$$

solving above equation, we get

$$C^2 = \frac{2GM}{(r_p + r_a)r_a r_p} \tag{3.25}$$

Again using Eq. (3.23) and value of C^2 from above equation in LHS of the Eq. (3.24), we get

$$E = \frac{1}{2}m\frac{2GMr_p^2}{r_a r_p(r_a + r_a)} - \frac{GMm}{r_a} \tag{3.26}$$

Solving above equation, and substituting $r_a + r_p = 2a$, we get

$$E = -\frac{GMm}{2a} \tag{3.27}$$

(c) **[1 marks]** Plot the kinetic energy (KE) of the satellite as a function of θ over one full orbit. Mark any significant points in terms of a, e, and other variables.

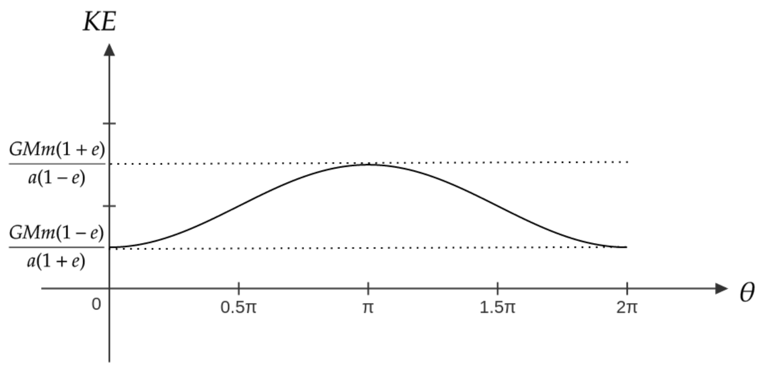
Solution:

$$KE = \frac{1}{2}mv^2 \tag{3.28}$$

where $v^2 = v_r^2 + v_t^2$. Substituting the value of v_r and v_t in the above equation, we get

$$KE = \frac{GMm(1 + e^2 - 2e\cos\theta)}{a(1 - e^2)} \tag{3.29}$$

For $\theta = 0$, $KE = KE_{min} = \frac{GMm(1-e)}{a(1+e)}$, and for $\theta = \pi$ $KE = KE_{max} = \frac{GMm(1+e)}{a(1-e)}$



(d) **[1.5 marks]** The perigee and apogee of the elliptical orbit in part (a) are 200 km and 36500 km, respectively. It is generally described as a (200×36500) km orbit. Here the distances are defined from the surface of the Earth. Calculate the period of rotation T (in hr) of

Chandrayaan-3 in this orbit.

Solution: The period of the orbit is given by

$$T^2 = \frac{4\pi^2}{GM}a^3 \tag{3.30}$$

where $a = \frac{r_p+r_a}{2}$, where r_p is perigee distance given by $r_p = 200\text{km} + R_E$ and r_a is apogee distance given by $r_a = 18350\text{km} + R_E$ and i.e. $a = 6371 + \frac{(200+36500)\text{km}}{2} = 24721\text{km}$

$$T = \sqrt{\frac{4\times\pi^2}{6.67\times10^{-11}\times5.972\times10^{24}}24721^3\times10^9} \tag{3.31}$$

$$= 10.75 \text{ hr} \tag{3.32}$$

(e) **[2.5 marks]** To move *Chandrayaan-3* from the first orbit (in part (d)) to another elliptical orbit EBN-1, an instantaneous boost was applied at perigee by changing the velocity by Δv, without altering the direction. This changed the apogee to 41800 km above Earth's surface while keeping the perigee unchanged. Calculate Δv.

Solution: since the velocities at P and A are purely tangential, and P is closer to earth as compared to A, hence the tangential speed is maximum at P. Let the velocity of the satellite in the given orbit at P be v_p and velocity at A be v_a. Conservation of energy gives:

$$\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = \frac{1}{2}mv_p^2 - \frac{GMm}{r_p} \tag{3.33}$$

Conservation of angular momentum gives

$$mv_ar_a = mv_pr_p \tag{3.34}$$

$$v_p = v_a\frac{r_a}{r_p} \tag{3.35}$$

Substituting the above equation into the energy conservation equation, we get

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p+r_a)}} \tag{3.36}$$

$$v_p = \sqrt{\frac{GMr_a}{ar_p}} \tag{3.37}$$

$$v_p = \sqrt{\frac{GM}{a}\frac{(1+e)}{(1-e)}} \tag{3.38}$$

Similarly

$$v_a = \sqrt{\frac{2GMr_p}{r_a(r_p+r_a)}} \tag{3.39}$$

$$v_a = \sqrt{\frac{GMr_p}{ar_a}} \tag{3.40}$$

$$v_a = \sqrt{\frac{GM}{a}\frac{(1-e)}{(1+e)}} \tag{3.41}$$

From Eq. (3.36)

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p+r_a)}} \tag{3.42}$$

$$v_p = 10.251 \text{ km/s} \tag{3.43}$$

In the new orbit, the perigee distance is kept the same, and the apogee distance changed to 41800 km, hence $r_a = 41800 + R_E = 48171$km and r_p remains as it is. Hence, the new velocity at the perigee distance is v'_p Using the expression Eq. (3.36), the v'_p can be written as

$$v'_p = \sqrt{\frac{2GMr_a}{r_p(r'_a + r_p)}} \tag{3.44}$$

$$v'_p = 10.327 \text{ km/s} \tag{3.45}$$

Now the boost required Δv at this position would be

$$\Delta v = v'_p - v_p \tag{3.46}$$

$$\Delta v = 0.076 \text{ km/s} \tag{3.47}$$

(f) **[1.5 marks]** After a series of manoeuvres, *Chandrayaan-3* was placed in an elliptical orbit of (100 × 1437) km around the Moon. Here, the distances are calculated from the surface of the Moon. Calculate the change in velocity $\Delta v'$, applied at the perigee, that is required to bring *Chandrayaan-3* from this elliptical orbit to a circular orbit at a distance of 100 km from the surface of the Moon. For this part, assume that *Chandrayaan-3* is only under the influence of the Moon's gravitational field.

Solution: Let the apogee distance be $r_{ma} = R_m + 1437$ km and perigee distance be $r_{mp} = R_m + 100$ km The velocity of the satellite at perigee, when it is in the elliptic orbit, is v_{mp} and the velocity when it is in the circular orbit is v_m we know that

$$v_{mp} = \sqrt{\frac{2GM_M r_{ma}}{r_{mp}(r_{ma} + r_{mp})}} \tag{3.48}$$

$$v_{mp} = 1.838 \text{ km/s} \tag{3.49}$$

Similarly here apogee distance be $r_{ma} = R_m + 100$ km and perigee distance be $r_{mp} = R_m + 100$ km. Let $r_m = r_{ma} = r_{mp}$

$$v_m = \sqrt{\frac{2GM_m r_m}{r_m(r_m + r_m)}} \tag{3.50}$$

$$v_m = 1.633 \text{ km/s} \tag{3.51}$$

$$\tag{3.52}$$

The boost required $\Delta v'$

$$\Delta v' = v_m - v_{mp} \tag{3.53}$$

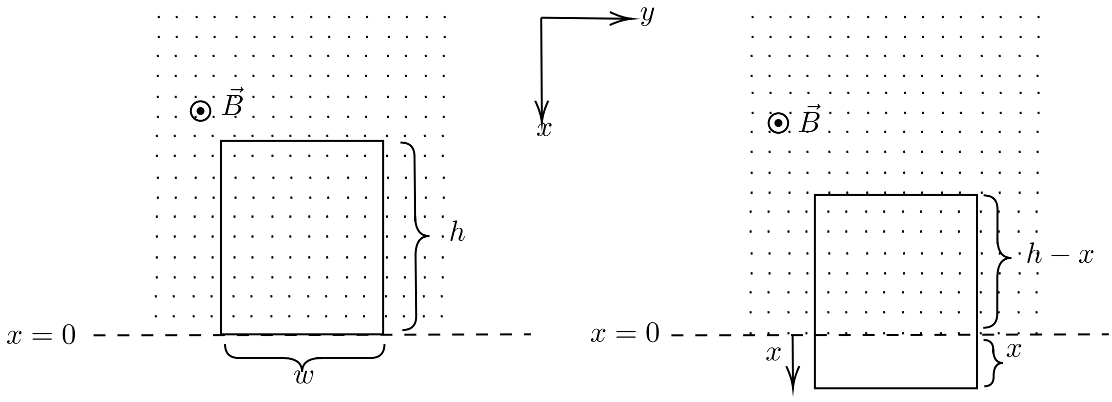
$$\Delta v' = -0.205 \text{ km/s} \tag{3.54}$$

4. **Mag-Grav Tussle**

A rectangular conducting loop of mass m, width w, length h, and self inductance L is held in the vertical x-y plane with its bottom edge along the y-axis (see figure on the left below). In this problem take the resistance of the loop to be zero. A uniform magnetic field $\vec{B}$ is applied horizontally as shown in the figure such that

$$\begin{aligned} \vec{B} &= B\hat{k} \quad \text{for } x \le 0 \\ &= 0 \quad \text{for } x > 0 \end{aligned}$$

The loop is released from rest at time $t = 0$ and descends under gravity (see the figure to the right below). The acceleration due to gravity g is in $+x$ direction.



(a) (b)

(a) **[5 marks]** Obtain $x(t)$, the position of the bottom edge of the loop at time t, in terms of relevant variables.

Solution:

$$m\ddot{x} = mg - BIw \tag{4.1}$$

$$\phi = Bw(h - x) + LI \tag{4.2}$$

$$-IR = \dot{\phi} = -Bw\dot{x} + L\dot{I} \tag{4.3}$$

Since $R = 0$ Implies

$$\dot{I} = \frac{Bw\dot{x}}{L} \tag{4.4}$$

Differentiating equation of motion with respect to t

$$m\ddot{v} = -Bw\dot{I} \tag{4.5}$$

$$\ddot{v} = \frac{-B^2w^2\dot{x}}{mL} \tag{4.6}$$

$$= -\omega_0^2 v \tag{4.7}$$

where

$$\omega_0^2 = \frac{B^2w^2}{mL} \tag{4.8}$$

The solution to v is

$$v = A\cos\omega_0 t + D\sin\omega_0 t \tag{4.9}$$

$$\dot{v} = -A\omega_0\sin\omega_0 t + D\omega_0\cos\omega_0 t \tag{4.10}$$

$$\ddot{v} = -A\omega_0^2\cos\omega_0 t + D\omega_0^2\sin\omega_0 t \tag{4.11}$$

Applying boundary conditions at $t = 0$, $\dot{v} = g$, $v = 0$ which implies that

$$D = \frac{g}{\omega_0} \tag{4.12}$$

$$A = 0 \tag{4.13}$$

Hence,

$$v = \frac{g}{\omega_0}\sin\omega_0 t \tag{4.14}$$

$$x = -\frac{g}{\omega_0^2}\cos\omega_0 t + C \tag{4.15}$$

at $t = 0$, $x = 0$, which implies that $C = \dfrac{g}{\omega_0^2}$

Hence,

$$x = \frac{g}{\omega_0^2}(1 - \cos\omega_0 t) \tag{4.16}$$

(b) **[6 marks]** Imagine different possible scenarios for the nature of motion of the loop and plot $x(t)$ for each.

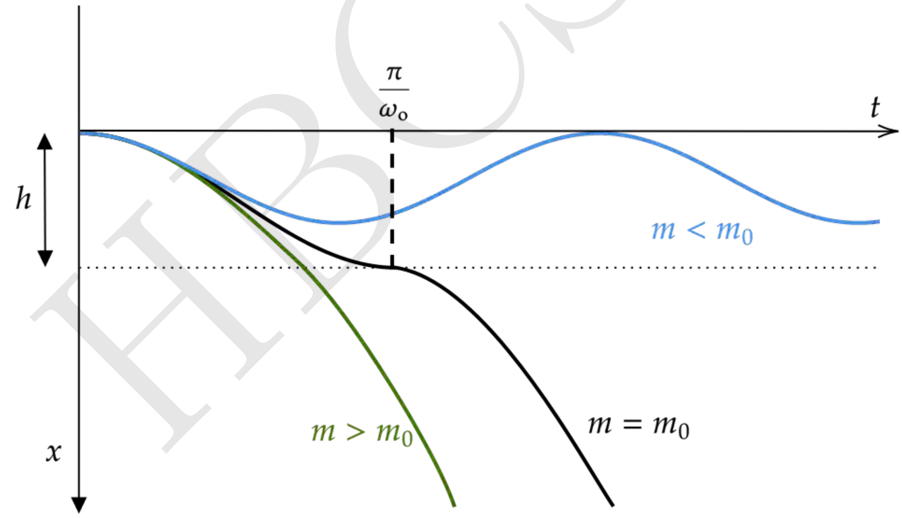
Solution: We found that

$$x = \frac{g}{\omega_0^2}(1 - \cos\omega_0 t) \tag{4.17}$$

The frequency of the oscillation is inversely proportional to m, and the amplitude increases with m. When the loop oscillates, the amplitude is

$$x_m = 2\frac{g}{\omega_0^2} = 2\frac{gmL}{B^2w^2} \tag{4.18}$$

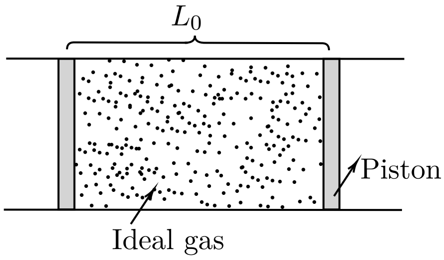
We can take three limiting cases, $m > m_0, m = m_0$, and $m < m_0$, where $m_0 = hB^2w^2/2gL$.



For $m < m_0$, the loop oscillates. For $m = m_0$, the amplitude of the oscillation is h, and the loop comes out of the magnetic field region and falls under gravity. For $m > m_0$, the loop will come out of the magnetic field quicker it falls under gravity.

5. **Thermal Tussle**

Consider a horizontal insulated cylindrical tube of very large length. Two identical insulated pistons, each of mass $M = 0.2$ kg are fitted within the tube separated by a length $L_0 = 1$ m. The space between the two pistons is filled with one mole of (ideal) helium gas, initially at temperature $T_0 = 300$K. The external pressure, everywhere outside the pistons and tube, is zero.



Initially, the pistons are held in place by an external mechanism. At time $t = 0$, the mechanism is released and the pistons move without friction and the process is quasistatic initially. Assume that the gas behaves ideally throughout. Let C_p and C_v be the specific heats of the gas at constant pressure and volume respectively. Also, $\gamma = C_p/C_v = 5/3$.

(a) **[6 marks]** Determine the velocity (v_p) of each piston in terms of the gas temperature T and other relevant variables. At what temperature (T_c), is the process no longer quasistatic? Calculate T_c.

Solution: Given the initial temperature of the system to be T_0, the initial energy of the system is C_vT_0. When the piston starts moving, from the work-energy theorem, the energy of the system is

$$\frac{1}{2}Mv_1^2 + \frac{1}{2}Mv_2^2 + C_vT = C_vT_0 \tag{5.1}$$

Also from conservation momentum, we have $v_2 = -v_1 = v$.
From above equation, we get

$$Mv^2 + C_vT = C_vT_0 \tag{5.2}$$

$$v^2 = \frac{C_v}{M}(T_0 - T) \tag{5.3}$$

$$v = \sqrt{\frac{C_v}{M}(T_0 - T)} \tag{5.4}$$

For the process to be quasistatic and adiabatic the piston's velocity cannot be greater than rms velocity of the gas.

$$v < v_{\text{rms}} \tag{5.5}$$

$$\sqrt{\frac{C_v}{M}(T_0 - T)} < \sqrt{\frac{3RT}{m}} \tag{5.6}$$

where m is molar mass of the gas. Solving the above equation, we get

$$T > \frac{C_vT_0m}{3RM + mC_v} \approx 3\text{K} \tag{5.7}$$

Below this temperature, the piston's velocity exceeds rms velocity, which indicates that the piston moves very rapidly. This is where the quasi-static limit will breaks down. For the estimation purpose, we can also take the average velocity or the most probable velocity and the corresponding limit would be 3.5K and 4.4K respectively.

(b) **[4 marks]** From here, we restrict our analysis only to the quasistatic regime of the process. We define $u = T/T_0$. Obtain the relation between u and t in the following form

$$t = f(u)$$

You may leave the answer in terms of a suitable integral involving L_0, M and other variables.

Solution: Since the process adiabatic.

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \tag{5.8}$$

Which implies

$$TL^{\gamma-1} = T_0L_0^{\gamma-1} \tag{5.9}$$

To express the temperature as a function of time, Differentiating Eq. (5.8) w.r.t t, we get

$$L^{\gamma-1}\frac{dT}{dt} + T(\gamma-1)L^{\gamma-2}\frac{dL}{dt} = 0 \tag{5.10}$$

From Eq. (5.8), we get

$$L = \left(\frac{T_0L_0^{(\gamma-1)}}{T}\right)^{\frac{1}{\gamma-1}} \tag{5.11}$$

Also

$$\frac{dL}{dt} = 2v = 2\sqrt{\frac{C_v}{M}(T_0-T)} \tag{5.12}$$

Substituting Eq. (5.11) and Eq. (5.12) into Eq. (5.10), we get

$$\frac{T_0L_0^{(\gamma-1)}}{T}\frac{dT}{dt} + T(\gamma-1)\left(\frac{T_0L_0^{(\gamma-1)}}{T}\right)^{(\frac{\gamma-2}{\gamma-1})} 2\sqrt{\frac{C_v}{M}(T_0-T)} = 0 \tag{5.13}$$

For Mono atomic gas $\gamma = 5/3$, hence above equation becomes

$$\frac{T_0L_0^{2/3}}{T}\frac{dT}{dt} + T\frac{2}{3}\left(\frac{T_0L_0^{2/3}}{T}\right)^{-1/2} 2\sqrt{\frac{C_v}{M}(T_0-T)} = 0 \tag{5.14}$$

Rearranging above equation, we get,

$$dt = -\frac{1}{B_1}\frac{dT}{T^{5/2}(T_0-T)^{1/2}} \tag{5.15}$$

Where $B_1 = \dfrac{2\sqrt{2/3}\sqrt{N_Ak/M}}{L_0T_0^{3/2}}$ Integrating above equation, we get

$$\int_0^t dt = -\int \frac{1}{B_1}\frac{\frac{dT}{T_0}}{(\frac{T}{T_0})^{5/2}T_0^2(1-\frac{T}{T_0})^{1/2}} \tag{5.16}$$

Let $u = T/T_0$, then above integral becomes

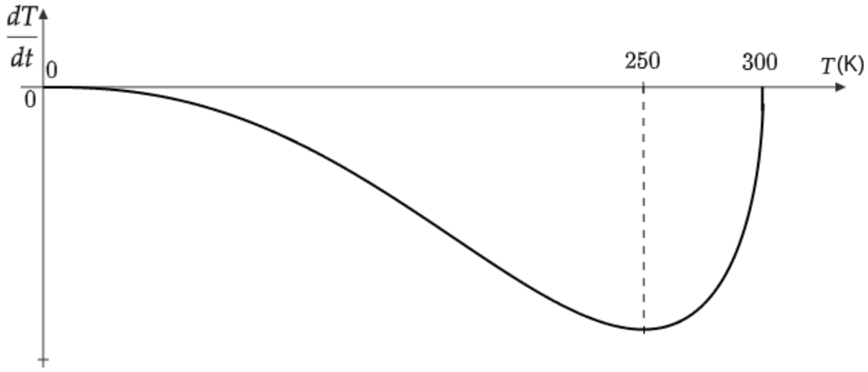
$$\int_0^t dt = -\int_{u_0}^{u} \frac{1}{B_1}\frac{du}{u^{5/2}T_0^2(1-u)^{1/2}} \tag{5.17}$$

$$t = -\frac{1}{B_1T_0^2}\int_{u_0}^{u}\frac{du}{u^{5/2}(1-u)^{1/2}} \tag{5.18}$$

(c) **[4 marks]** Qualitatively plot the rate of change of temperature (dT/dt) vs T. Mark any significant point(s) on the temperature axis in the plot.

Solution: Temperature decreases over time. From Eq. (5.15), it is evident that the derivative of the temperature function is always negative. Additionally, at $t = 0$ and

$t = 300$ K, $dT/dt = 0$. The function exhibits an extremum at $T = 250$ K. These details are illustrated in the figure below.



(d) **[4 marks]** At what time t does the temperature T of the gas reach 20K? What is the piston velocity (v_{p}) at this point?

Solution: The integration from Eq. (5.18) can be solved by substituting $u = \cos^2\theta$ and using boundary conditions as $u = 1$ for $T = T_0$, we get

$$\left(\frac{1-u}{u}\right)^{3/2} + 3\left(\frac{1-u}{u}\right)^{1/2} - \frac{3B_1T_0^2t}{2} = 0 \tag{5.19}$$

Using above equation, for $n = 1$ moles and $L_0 = 1$ m, the temperature reaches 20K after 0.165s.
From Eq. (5.4) The piston's velocity at this point is 132.1 m/s.

6. **Sonic Sleuth**
During her summer vacation, Dheera decides to carry out a smartphone based experiment. She utilizes a smartphone's frequency sensor that can measure the frequency of the audio signal it receives. She takes a long cylindrical tube closed at one end. This tube has a length of $L = 30.0$ cm and an inner diameter of $d = 2.45$ cm. Dheera starts filling the tube with water, which is dripping from a tap at a constant rate Q (measured in milliliters per second (mL/s)).

Dheera positions her smartphone near the open end of the tube to measure the frequency of the sound emitted as water fills the tube. An app on the phone captures a range of frequencies in the recorded audio at any given time. At randomly chosen values of time t, one of the frequencies at that time is shown in the following table.

Time t (s)	Frequency f (Hz)	Time t (s)	Frequency f (Hz)
5.0	915	36.0	434
7.6	320	39.6	481
16.2	345	41.9	500
16.7	1008	42.5	1454
20.9	360	51.1	1618
25.7	1148	51.6	574
28.9	1196	56.1	1782
31.5	410	60.2	680
33.3	1290	66.3	820

Help her to analyse the experiment.

(a) **[3 marks]** Derive the expression for the velocity of sound c_s in terms of f, t, and constants.

Solution: The frequency of the sound in the tube is determined by the formula:

$$f = \frac{nc_s}{4(h + 0.3d)} \tag{6.1}$$

Here, h and d represent the length of the air column and the diameter of the tube, respectively. The variable n is an odd integer representing the fundamental, third, fifth harmonics, and so on. The speed of sound is denoted by c_s, and the term $0.3d$ in the denominator accounts for the end correction in the tube.

When water falls at a constant rate (Q), it creates the disturbances in the air column of the tube. These disturbances travel as sound waves through the air column, which we detect and analyze. If the length of the tube is L, the equation (6.1) can be modified as:

$$c_s = \frac{f4(L - (\frac{Q}{A})t) + 0.3d)}{n} \tag{6.2}$$

Here, $A = \pi d^2/4$ represents the area of the base, and n is an odd integer representing the fundamental, third, fifth harmonics, and so on.

(b) **[8 marks]** Choose a pair of suitable variables and plot a linear graph. Specify the axis labels. Obtain the speed of sound c_s and the rate Q from this plot.

Solution: Since the height of the water level varies linearly with time, we expect the frequency of the sound to increase with time. By linearizing the equation (6.1), we get:

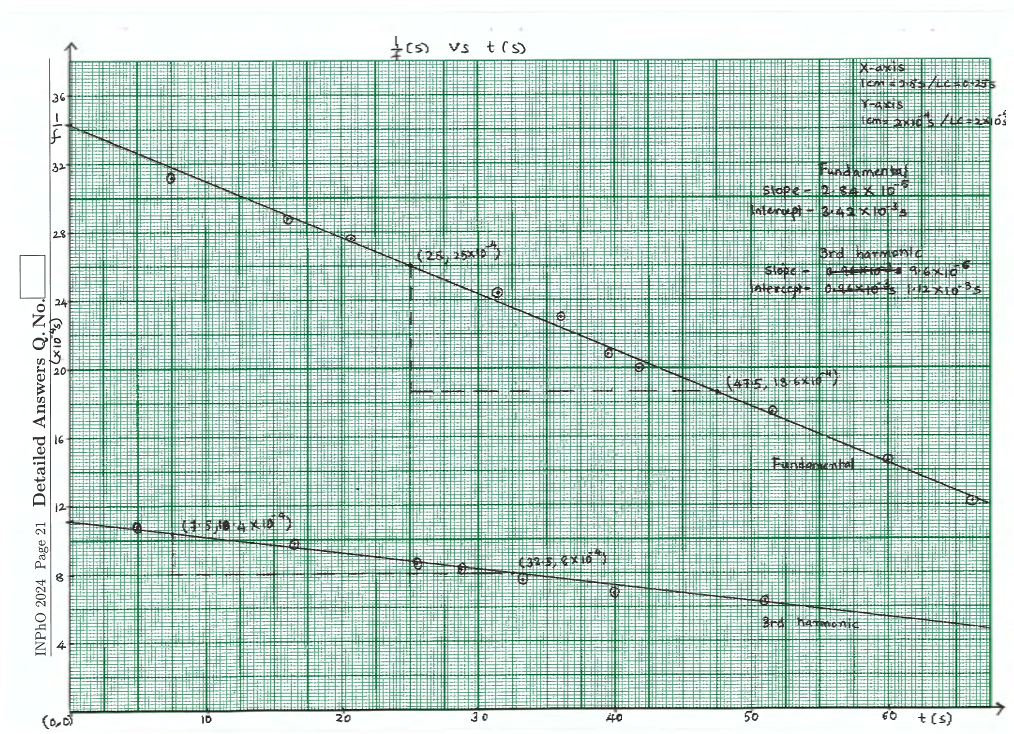
$$f = \frac{nc_s}{4(L - (\frac{Q}{A})t) + 0.3d)} \tag{6.3}$$

$$\frac{1}{f} = -\frac{4(\frac{Q}{A})t}{nc_s} + \frac{4(L + 0.3d)}{nc_s} \tag{6.4}$$

Plotting the relationship between $1/f$ and t will yield a linear graph. The values for the plot are as follows.

Fundamental			3rd Harmonic		
Time(s)	Frequency(Hz)	1/f (s)	Time(s)	Frequency(Hz)	1/f (s)
7.6	320	0.00313	5	915	0.00109
16.2	345	0.00290	16.7	1008	0.00099
20.9	360	0.00278	25.7	1148	0.00087
31.5	410	0.00244	28.9	1196	0.00084
36	434	0.00230	33.3	1290	0.00078
39.6	481	0.00208	42.5	1454	0.00069
41.9	500	0.00200	51.1	1618	0.00062
51.6	574	0.00174	56.1	1782	0.00056
60.2	680	0.00147			
66.3	820	0.00122			

The presence of two distinct straight lines in the graph indicates the existence of two harmonic frequencies in the dataset. Upon examining the ratio of these frequencies, it becomes evident that these two lines correspond to $n = 1$ and $n = 3$. The intercept and slope of the graph can be used to determine the speed of sound and the rate of water filling, respectively.



	Fundamental	3rd Harmonic
Slope	-2.84×10^{-5}	-1.12×10^{-5}
Intercept	3.42×10^{-3}	1.12×10^{-3}
C_s	359 m/s	366 m/s
Q	1.20 mL/s	1.24 mL/s

****** END OF THE QUESTION PAPER ******

Space for rough work — will NOT be submitted for evaluation