Solution Problem 1

Eclipses of the Jupiter's Satellite

a. (Total Point : 1) Assume the orbits of the earth and Jupiter are circles, we can write the centripetal force = equal gravitational attraction of the Sun.

$$G\frac{M_E M_s}{R_E^2} = \frac{M_E V_E^2}{R_E}$$

$$G\frac{M_J M_S}{R_J^2} = \frac{M_J V_J^2}{R_J}$$

(0.5 point)

where

G = universal gravitational constant
M_S = mass of the Sun
M_E = mass of the Earth
M_J = mass of the Jupiter
R_E = radius of the orbit of the Earth
V_E = velocity of the Earth
V_J = velocity of Jupiter

Hence

$$\frac{R_J}{R_E} = \left(\frac{v_E}{v_J}\right)^2$$

We know

$$T_E = \frac{2\pi}{\omega_E} = \frac{2\pi R_E}{v_E}, and$$

$$T_J = \frac{2\pi}{\omega_J} = \frac{2\pi R_J}{v_J}$$

we get

$$\frac{T_E}{T_J} = \frac{\frac{R_E}{v_E}}{\frac{R_J}{v_J}} = \left(\frac{R_E}{R_J}\right)^{3/2}$$

$$R_J = 779.8 \times 10^6 \, km$$

(0.5 point)

b. **(Total Point: 1)** The relative angular velocity is

$$\omega = \omega_E - \omega_J = 2\pi\left(\frac{1}{365} - \frac{1}{11.9\times 365}\right)$$
$$= 0.0157 \, rad / day$$

(0.5 point)

and the relative velocity is

$$v = \omega R_E = 2.36\times 10^6 km / day$$
$$= 27.3\times 10^3 \, km$$

(0.5 point)

c. **(Total Point: 3)** The distance of Jupiter to the Earth can be written as follows

$$\mathrm{d}(t) = \mathrm{R}_J - \mathrm{R}_E$$
$$\mathrm{d}(t).\mathrm{d}(t) = (\mathrm{R}_J - \mathrm{R}_E).(\mathrm{R}_J - \mathrm{R}_E)$$

(1.0 point)

$$\mathrm{d}(t) = \left(\mathrm{R}_J^2 + \mathrm{R}_E^2 - 2R_E R_J \cos\omega t\right)^{\frac{1}{2}}$$
$$\approx R_J\left(1 - 2\left(\frac{R_E}{R_J}\right)\cos\omega t + \ldots\right)^{\frac{1}{2}}$$
$$\approx R_J\left(1 - \frac{R_E}{R_J}\cos\omega t + \ldots\right)$$

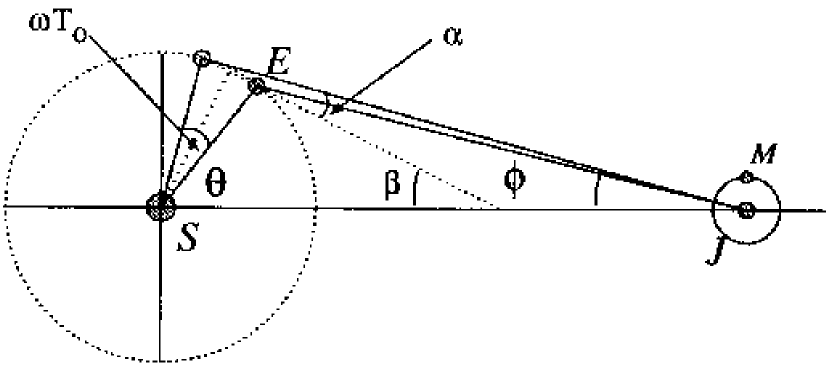

Figure 1: Geometrical relationship to get $\Delta d(t)$

The relative error of the above expression is the order of

$$\left(\frac{R_E}{R_J}\right)^2 \approx 4\%$$

The observer saw M begin to emerge from the shadow when his position was at *d(t)* and he saw the next emergence when his position was at *d(t +T0)*/ Light need time to travel the distance $\Delta d = d(t+T_0)-d(t)$ so the observer will get apparent period T instead of the true period T_o.

$$\Delta d = R_E(\cos\omega t - \cos\omega(t+T_0)) \approx R_E\omega T_0 \sin\omega t$$

(1.0 point)

because $\omega T_0 \approx 0.03, \sin\omega t + ..., \cos\omega T_0 \approx 1 - ...$

We can also get this approximation directly from the geometrical relationship from Figure 1.

(1.0 point)

or we can use another method.

From the figure above we get

$$\beta = (\phi + \alpha)$$
$$\frac{\omega T_0}{2} + \beta + \theta = \frac{\pi}{2}$$

(1.0 point)

$$\Delta d \approx \omega T_0 R_E \cos\alpha$$
$$\approx \omega T_0 R_E \sin\left(\omega t + \frac{\omega T_0}{2} + \phi\right)$$
$$\omega T_0 \approx 0.03 \text{ and } \phi \approx 0.19$$

(1.0 point)

d. (Total Point: 2)

$$T - T_0 \approx \frac{\Delta d(t)}{c}; c = \text{velocity of light}$$
$$T \approx T_0 + \frac{\Delta d(t)}{c} = T_0 + \frac{R_E \omega T_0 \sin\omega t}{c}$$

(1.0 point)

e. Total Point : 2 from

$$T_{\max} = T_0 + \frac{R_E \omega T_0}{c}$$

we get

$$\frac{R_E \omega T_0}{c} = 15$$

Hence

C=2.78 x 10^5 km/s

(1.0 point)