Question 3 - Marking Scheme

(a) Since $W(v) = 4\pi \left(\dfrac{M}{2\,\pi\,R\,T} \right)^{3/2} v^2 \; e^{-M v^2/(2RT)}$,

$$\bar{v} = \int_0^\infty v \; W(v)\, dv =$$

$$= \int_0^\infty v \;\; 4\pi \left(\frac{M}{2\,\pi\,R\,T} \right)^{3/2} v^2 \; e^{-M v^2/(2RT)} \; dv$$

$$= \int_0^\infty 4\pi \left(\frac{M}{2\,\pi\,R\,T} \right)^{3/2} v^3 \; e^{-M v^2/(2RT)} \; dv$$

$$= \; 4\pi \left(\frac{M}{2\,\pi\,R\,T} \right)^{3/2} \int_0^\infty v^3 \; e^{-M v^2/(2RT)} \; dv$$

$$= 4\pi \left(\frac{M}{2\,\pi\,R\,T} \right)^{3/2} \frac{4\,R^2\,T^2}{2\,M^2}$$

$$= \sqrt{\frac{8\,R\,T}{\pi\,M}}$$

Marking Scheme:

Performing the integration correctly:	1 mark
Simplifying	0.5 marks

Subtotal for the section 1.5 marks

(b) Assuming an ideal gas, $PV = NkT$, so that the concentration of the gas molecules, n, is given by

$$n = \frac{N}{V} = \frac{P}{kT}$$

the impingement rate is given by

$$J = \frac{1}{4} n \bar{v}$$

$$= \frac{1}{4} \frac{P}{kT} \sqrt{\frac{8RT}{\pi M}}$$

$$= P \sqrt{\frac{8RT}{16 k^2 T^2 \pi M}}$$

$$= P \sqrt{\frac{N_A k}{2 k^2 T \pi M}}$$

$$= P \sqrt{\frac{1}{2 k T \pi m}}$$

$$= \frac{P}{\sqrt{2 \pi m k T}}$$

where we have note that $R = N_A k$ and $m = \frac{M}{N_A}$ (N_A being Avogadro number).

Marking Scheme:

Using ideal gas formula to estimate concentration of gas molecules: 0.7 marks

Simplifying expression: 0.4 marks

Using $R = N k$, and the formula for *m;* (0.2 mark each) 0.4 marks

Subtotal for the section 1.5 marks

(c) Assuming close packing, there are approximately 4 molecules in an area of 16 r^2 m^2. Thus, the number of molecules in 1 m^2 is given by

$$n_1 = \frac{4}{16\,(3.6 \times 10^{-10})^2} = 1.9 \times 10^{18}\ \text{m}^{-2}$$

However at (273 + 300) K and 133 Pa, the impingement rate for oxygen is

$$J = \frac{P}{\sqrt{2\pi mkT}}$$

$$= \frac{133}{\sqrt{2\pi\left(\frac{32 \times 10^{-3}}{6.02 \times 10^{23}}\right)(1.38 \times 10^{-23})573}}$$

$$= 2.6 \times 10^{24}\ \text{m}^{-2}\,\text{s}^{-1}$$

Therefore, the time needed for the deposition is $\frac{\boldsymbol{n_1}}{\boldsymbol{J}} = 0.7\ \mu\text{s}$

The calculated time is too short compared with the actual processing.

Marking Scheme:

Estimation of number of molecules in 1 m^2 :	0.4 marks
Calculation the impingement rate:	0.6 marks
Taking note of temperature in Kelvin	0.3 marks
Calculating the time	0.4 marks

Subtotal for the section 1.7 marks

(d) With activation energy of 1 eV and letting the velocity of the oxygen molecule at this energy is v_1, we have

$$\frac{1}{2} m v_1^2 = 1.6 \times 10^{-19} \text{ J}$$

$$\Rightarrow v_1 = 2453.57 \quad \text{ms}^{-1}$$

At a temperature of 573 K, the distribution of the gas molecules is

We can estimate the fraction of the molecules with speed greater than 2454 ms^{-1} using the trapezium rule (or any numerical techniques) with ordinates at 2453, 2453 + 500, 2453 +1000. The values are as follows:

Velocity, v	Probability, $W(v)$
2453	1.373×10^{-10}
2953	2.256×10^{-14}
3453	6.518×10^{-19}

Using trapezium rule, the fraction of molecules with speed greater than 2453 ms^{-1} is given by

$$\text{fraction of molecules} = \frac{500}{2}\left[\left(1.373\times10^{-10}\right)+\left(2 \times 2.256\times10^{-14}\right)+\left(6.518\times10^{-19}\right)\right]$$

$$f = 3.43 \times 10^{-8}$$

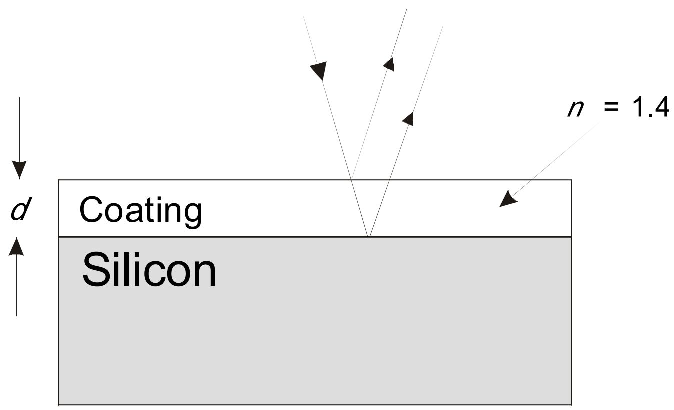
Thus the time needed for the deposition is given by 0.7 µs/(3.43 x 10^{-8}) that is 20.4 s

Marking Scheme

Computing the value of the cut-off energy or velocity: 0.6 marks
Estimating the fraction of molecules 1.2 marks
Correct method of final time 0.4 marks
Correct value of final time 0.6 marks

Subtotal for the section 2.8 marks

(e) For destructive interference, optical path difference $= 2\,d = \dfrac{\lambda'}{2}$ where $\lambda' = \dfrac{\lambda_{\text{air}}}{n}$ is the wavelength in the coating.



The relation is given by:

$$d = \frac{\lambda_{\text{air}}}{4\,n}$$

Plugging in the given values, one gets d = 105 or 105.2 nm.

Derive equation:

Finding the optical path length 0.2 marks
Knowing that there is a phase change at the reflection 0.5 marks
Putting everything together to get the final expression 0.6 marks
Subtotal: 1.3 marks

Computation of d: 0.6 marks
Getting the correct number of significant figures: 0.6 marks
Subtotal: 1.2 marks

Subtotal for Section **2.5 marks**

TOTAL **10 marks**