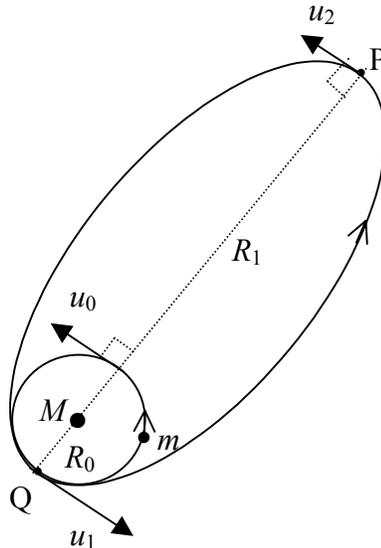


Theoretical Competition

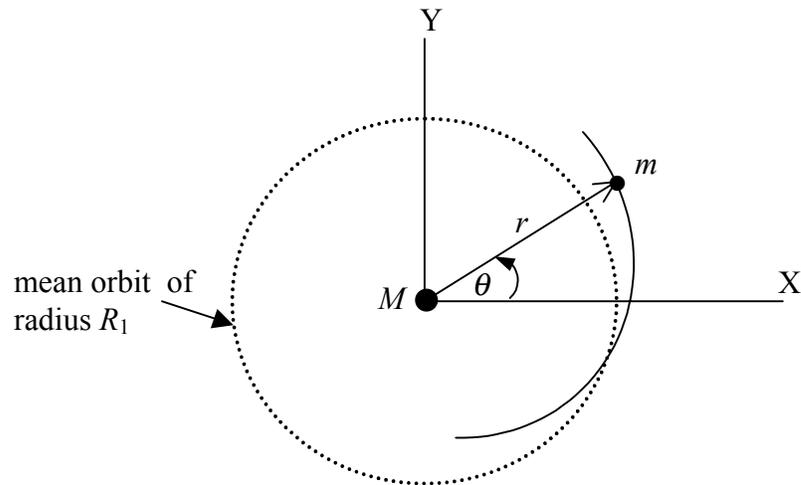
I. Satellite's orbit transfer

In the near future we ourselves may take part in launching of a satellite which, in point of view of physics, requires only the use of simple mechanics.



- A satellite of mass m is presently circling the Earth of mass M in a circular orbit of radius R_0 . What is the speed (u_0) of mass m in terms of M, R_0 and the universal gravitation constant G ? (1 point)
- We are to put this satellite into a trajectory that will take it to point P at distance R_1 from the centre of the Earth by increasing (almost instantaneously) its velocity at point Q from u_0 to u_1 . What is the value of u_1 in terms of u_0, R_0, R_1 ? (2 points)
- Deduce the minimum value of u_1 in term of u_0 that will allow the satellite to leave the Earth's influence completely. (1 point)
- (Referring to part b.) What is the velocity (u_2) of the satellite at point P in terms of u_0, R_0, R_1 ? (1 point)
- Now, we want to change the orbit of the satellite at point P into a circular orbit of radius R_1 by raising the value of u_2 (almost instantaneously) to u_3 . What is the magnitude of u_3 in terms of u_2, R_0, R_1 ? (1 point)

f)



If the satellite is slightly and instantaneously perturbed in the radial direction so that it deviates from its previously perfectly circular orbit of radius R_1 , derive the period of its oscillation T of r about the mean distance R_1 .

Hint: Students may make use (if necessary) of the equation of motion of a satellite in orbit:

$$m \left[\frac{d^2}{dt^2} r - \left(\frac{d}{dt} \theta \right)^2 r \right] = -G \frac{Mm}{r^2} \quad \dots \quad (1)$$

and the conservation of angular momentum:

$$mr^2 \frac{d}{dt} \theta = \text{constant} \quad \dots \quad (2)$$

(3 points)

g) Give a rough sketch of the whole perturbed orbit together with the unperturbed one.

(1 point)