Solution and Marking Scheme

Theory

II. Optical Gyroscope

The light wave moves with speed $c' = \frac{c}{\mu}$ in the medium having refractive index μ. Wavelength of light in medium $\lambda' = \frac{\lambda}{\mu}$, where λ is the wavelength of light in vacuum.

a) (2 points)

transit time for the CW beam: $t^{+} = \frac{2\pi R + R\Omega t^{+}}{c'} = \frac{2\pi R}{c'}(1 - \frac{R\Omega}{c'})^{-1}$

transit time for the CCW beam: $t^{-} = \frac{2\pi R - R\Omega t^{-}}{c'} = \frac{2\pi R}{c'}(1 + \frac{R\Omega}{c'})^{-1}$

the time difference between t^{+} and t^{-}: $\Delta t = \frac{4\pi R^2 \Omega}{(c')^2 - R^2\Omega^2}$

since $(R\Omega)^2 << (c')^2$ $\qquad \Delta t \approx \frac{4\pi R^2 \Omega}{(c')^2}$

b) (2 points) the round-trip optical path difference, ΔL, is given by

$$\Delta L = c'\Delta t = \frac{4\pi R^2 \Omega}{c'}$$

c) (1 point) $\Delta L \cong 4.5 \text{x} 10^{-12}$ m.

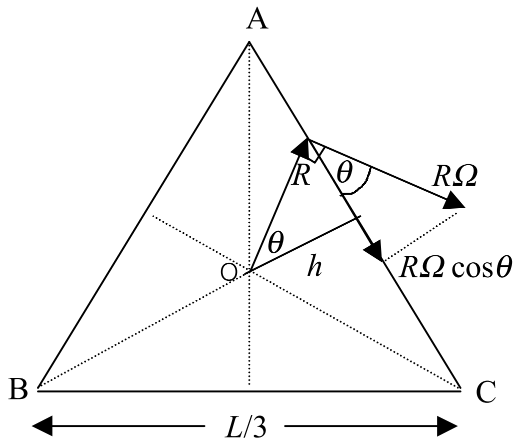
d) (1 point) the corresponding optical phase difference $\Delta\theta$ is,

$$\Delta\theta = \frac{2\pi \Delta L}{\lambda'} = \frac{8\pi^2 R^2 \Omega}{c\lambda'}, \text{ where } \lambda' = \frac{\lambda}{\mu}$$

for N turns of fiber optic ring,

$$\Delta\theta = \frac{8\pi^2 R^2 N\Omega}{c\lambda'}$$

e) (2 points)



The figure shows the triangular ring rotating about the centre o with the angular speed Ω in the clockwise direction. Without loosing generality, let's first consider the velocity of light along AC in the CW and CCW direction,

$v_{\pm} = c \pm R\Omega\cos\theta = c \pm \Omega h$, where h is constant.

$$\tau_{\pm} = \frac{L/3}{v_{\pm}} = \frac{L/3}{c \pm \Omega h} \approx \frac{L/3}{c}(1 \mp \frac{\Omega h}{c})$$

where $\tau_{\pm}$ is the time taken for light travelling along AC in the CW and CCW.

$t_{\pm} = \frac{L}{v_{\pm}} = \frac{L}{c \pm \Omega h} \approx \frac{L}{c}(1 \mp \frac{\Omega h}{c})$, where L is the perimeter of the triangular ring.

Therefore, the time difference of light travelling in one complete cycle.

$\Delta t = \frac{2\Omega L h}{c^2} = \frac{4\Omega}{c^2}\left(\frac{1}{2}Lh\right) = \frac{4\Omega A}{c^2}$, where A is the area of the triangular ring.

f) The resonance frequencies associated with $L_{\pm}$ corresponding to the effective cavity lengths seen by CW and CCW propagating beams respectively is,

$$L_+ = ct^+ \approx L(1 - \frac{\Omega h}{c})$$

$$L_- = ct^- \approx L(1 + \frac{\Omega h}{c})$$

where $L_{\pm}$ is the perimeter of the equilateral triangle in the CW (+) and CCW (-) and we also use the fact that $h\Omega << c$. Therefore,

$$\Delta L = L_- - L_+ = 2L\frac{\Omega h}{c} = \frac{4\Omega A}{c} = \frac{\Omega L^2}{\sqrt{3}\, c}$$

The condition to sustain the laser oscillation (given in the problem),

$\nu_{\pm} = \frac{m}{L_{\pm}}c$, m = 1, 2, 3, … integers (1 point)

$\Delta\nu = \nu_- - \nu_+ = \frac{m}{L_-}c - \frac{m}{L_+}c \approx mc\frac{\Delta L}{L^2} = \nu\frac{\Delta L}{L}$ (1 point) the approximation arises from $L_+ L_- \approx L^2$

where L is the perimeter of the triangular ring. Hence,

$$\Delta\nu = \frac{\Delta L}{L}\nu = \frac{4A}{Lc}\nu\Omega = \frac{1}{\sqrt{3}}\frac{L}{\lambda}\Omega \qquad \text{(1 point)}$$