Solution and Marking Scheme

Theory

III. Plasma Lens

a) Consider cylindrical Gaussian surface of radius r and length ℓ about the central axis.
From Gauss's law

$$E2\pi r\ell = \frac{1}{\varepsilon_0}(\text{charge inside}) \qquad (0.5 \text{ points})$$

$$= -\frac{1}{\varepsilon_0} ne(\pi r^2 \ell)$$

$$E_r = -\frac{ner}{2\varepsilon_0} \qquad (0.5 \text{ point})$$

b) From Ampere's law, $B_\theta = -\dfrac{\mu_0 nerv}{2}$ (2 points)

c) The net Lorentz force is

$$\vec{F} = \left(\frac{ne^2 r}{2\varepsilon_0} - \frac{\mu_0 ne^2 rv^2}{2}\right)\hat{r} = \frac{ne^2 r}{2\varepsilon_0}\left(1 - \frac{v^2}{c^2}\right)\hat{r}$$

where $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$ (1 point)

d) $F_r \to 0$ as $v \to c$, this implies the electric force and magnetic force cancel each other out. (1 point)

e) The stationary plasma particles have $v = 0$, hence $F_{r'} = \pm eE_{r'}$

where $E_{r'} = -\dfrac{neR^2}{2\varepsilon_o r'} + \dfrac{n_o er'}{2\varepsilon_o}$

for positive ion $F_{r'} = -\dfrac{ne^2R^2}{2\varepsilon_o r'} + \dfrac{n_o e^2 r'}{2\varepsilon_o}$

for electron $F_{r'} = \dfrac{ne^2R^2}{2\varepsilon_o r'} - \dfrac{n_o e^2 r'}{2\varepsilon_o}$

and there is no cancellation from the magnetic force.
As a result the plasma electrons will be blown out, and the ions are pulled in.
(2 points)

f) The net force on the electron beam in plasma medium is given by,

$$\vec{F} = \frac{ne^2 r}{2\varepsilon_0}\left(1 - \frac{v^2}{c^2}\right)\hat{r} - \frac{n_0 e^2 r}{2\varepsilon_0}\hat{r} \qquad (2 \text{ points})$$

in the limit $v \to c$, $\vec{F} \approx -\dfrac{n_0 e^2 r}{2\varepsilon_0}\hat{r}$ (1 point)