Solution of Problem No. 1

Measure the mass in the weightless state

1/ Formula for period of oscillation :

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m_0}{k}} \qquad (1)$$

One can deduce :

$$m_0 = \frac{k}{4\pi^2}T_0^{\,2} = 25{,}21 \text{ kg} \qquad (2)$$

2/ When the craft orbits the Earth, oscillating system is the spring with one end attached to the chair of mass m_0 and other end attached to he craft with mass M. This system oscillates like an object with mass

$$m_0' = \frac{m_0 M}{m_0 + M} \qquad (3)$$

attached to an end of the spring, other end of the spring is fixed (m_0' is the reduced mass of the system craft-chair). The period T_0' of the system is also given by (1) and (2). We can deduce :

$$\frac{m_0}{m_0'} = (\frac{T_0}{T_0'})^2 = (\frac{1{,}28195}{1{,}27395})^2 \qquad (4)$$

The mass M of the craft can be calculate from (3) :

$$M = \frac{m_0}{\frac{m_0}{m_0'} - 1} = \frac{25{,}21}{(\frac{T_0}{T_0'})^2 - 1} = \frac{25{,}21}{(\frac{1{,}28195}{1{,}27395})^2 - 1} = 2001 \text{ kg} \qquad (5)$$

Let m be the mass of astronaut and chair, the coressponding reduce mass is m':

$$m' = \frac{mM}{m + M} \qquad (6)$$

the expression of m is then :

$$m = \frac{m'}{1 - \frac{m'}{M}}$$

The reduce mass m' can be calculated from the oscillation period T' by using formula (2) :

$$m' = \frac{605{,}6}{4}.(\frac{2{,}33044}{3{,}1416})^2 = 83{,}31 \text{ kg}$$

the true value of the mass m is :

$$m = \frac{83{,}31}{1 - \frac{83{,}31}{2001}} = 86{,}93 kg$$

the true value of the mass of astronaut :

$$86{,}93 - 25{,}21 = 61{,}72 \text{ kg}$$