Solution of Problem No. 3
Compression and expansion of a two gases system

1.

a. The isotherm curve is shown in Figure 1.

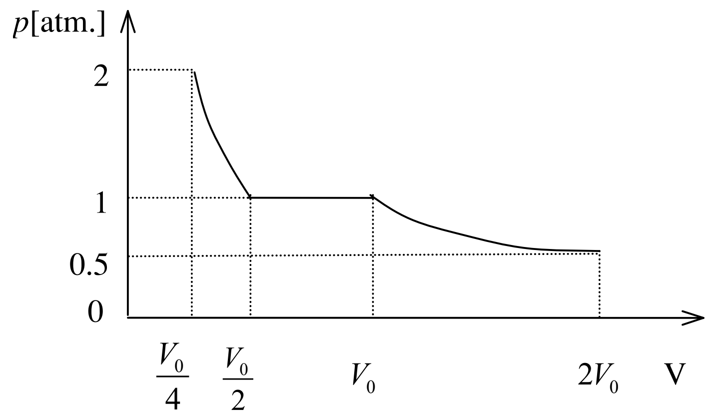

Figure 1

$$V_0 = \frac{RT_1}{p_1} = \frac{8.31\times 373}{0.5\times 1.013\times 10^5} = 0.0612\ \text{m}^3$$
$$= 61.2\ \text{dm}^3$$

b. The process of compressing can be divided into 3 stages:

$$\underset{(1)}{(p_1, 2V_0)} \rightarrow \underset{(2)}{(2p_1, V_0)} \rightarrow \underset{(3)}{(2p_1, V_0/2)} \rightarrow \underset{(4)}{(4p_1, V_0)}$$

The work in each stage can be calculated as follows:

$$A_{12} = -\int_{2V_0}^{V_0} p\,dV = 2RT_1 \int_{V_0}^{2V_0} \frac{dV}{V} = 2RT_1 \ln 2 = 4297\ \text{J}$$

$$A_{23} = 2p_1(V_0 - V_0/2) = RT_1 = 3100\ \text{J}$$

$$A_{34} = -\int_{\frac{1}{2}V_0}^{\frac{1}{4}V_0} p'\,dV = RT_1 \int_{\frac{1}{4}V_0}^{\frac{1}{2}V_0} \frac{dV}{V} = RT_1 \ln 2 = 2149\ \text{J}.$$

The total work of gases compressing is:

$$A = A_{12} + A_{23} + A_{34} = 9545\ \text{J} \cong 9.55\ \text{kJ} \qquad (1)$$

c. In the second stage 23, all the water vapor (one mole) condenses. The heat Q' delivered in the process equals the sum of the work A and the decrease ΔU of internal energy of one mole of water vapor in the condensing process.

$$Q' = \Delta U + A_{12} + A_{23} + A_{34}$$

One can remark that $\Delta U + A_{23}$ is the heat delivered when one mole of water vapor condenses, and equals $0.018\times L$. Thus

$$Q' = \Delta U + A_{11}+A_{23}+A_{34} = 0.018\times L + A_{11} + A_{34} = 46.946 \text{ J} \cong 47 \text{ kJ} \qquad (2)$$

2. The process of compression (2. a.) and expansion (2.c.) of gases can be divided into several stages. The stages are limited by the following states:

State	Left compartment Volume | Pressure | (atm.)	Right compartment Volume | Pressure | (atm.)	Total volume	Pressure on piston (atm.)
1	V_0 | 0.5	V_0 | 0.5	$2V_0$	0.5
2	V_0 | 0.5	$0.5\,V_0$ | 1	$1.5V_0$	1
3	$0.5V_0$ | 1	$V_0/3$ | 1.5	$5/6\,V_0$	1.5
4	0 | 1	$V_0/3$ | 1.5	$V_0/3$	1.5
5	0 | 1.5	$V_0/4$ | 2	$V_0/4$	2
6	0 | 1.5	$V_0/3$ | 1.5	$V_0/3$	1.5
7	0 | 1	V_0 | 0.5	V_0	0.5
8	$0.5V_0$ | 1	V_0 | 0.5	$1.5V_0$	0.5
9	$V_0(2-\sqrt{2}\,)$|$(\sqrt{2}+2)/4$	$V_0\sqrt{2}$ | $\sqrt{2}/4$	$2V_0$	$\sqrt{2}/4 = 0.35$

a. See figure 2 below.

b. The work A_p done by the piston in the process of compressing the gases equals the sum of the work A calculated in 1. and the work done by the force of friction. The latter equals 0.5 atm.$\times V_0 = p_1V_0$ (the force of kinetic friction appears in the process 234 during which the displacement of the partition NM corresponding to a variation V_0 of volume of the left compartment). Then, we have:

$$A_p = A + p_1V_0 = 9545 + 8.31 \times 373 = 12645 \text{ J} \cong 12.65 \text{ kJ}$$

c. In process 89 the pressure in the left compartment is always larger than the pressure in the right one (with a difference of 0.5 atm.). If p denotes the pressure in the right compartment, the pressure in the left one will be $p + 0.5$ atm.

Let V be the total volume in process 89, we have

$$\frac{RT_1}{p} + \frac{RT_1}{p+0.5} = V$$

with $V = 2V_0 = \dfrac{2RT_1}{0.5}$, p can be defined by $\dfrac{RT_1}{p} + \dfrac{RT_1}{p+0.5} = \dfrac{2RT_1}{0.5}$

This is equivalent to:

$$p + 0.5 + p = 4p.(p+0.5)$$

$$p = \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4} = 0.35 \text{ atm.}$$

The pressure in the right compartment is $p = 0.35$ atm.

The volume of the right compartment is $\sqrt{2}\ V_0$

The pressure in the left compartment is $p + 0.35 = 0.85$ atm.

The volume of the left compartment is $(2 - \sqrt{2}\,)\,V_0$

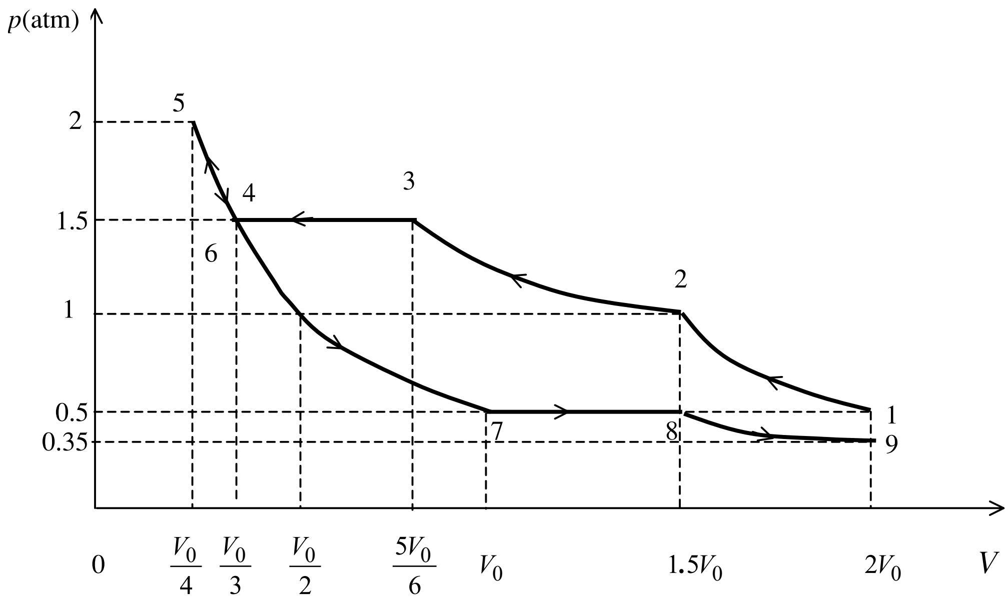

Figure 2

3.

a. Apply the first law of thermodynamic for the system of two gases in the cylinder:

$$\delta q = dU + \delta A \qquad (3)$$

In an element of process in which the variations of temperature and volume are respectively dT and dV:

$$\delta q = 0 \quad ; \quad \mathrm{d}U = (C_{V1} + C_{V2})\mathrm{d}T = (\frac{R}{\gamma_1 - 1} + \frac{R}{\gamma_2 - 1})\mathrm{d}T \quad ; \quad \delta A = p.\mathrm{d}V$$

On the other hand: $pV = RT$ (4)

One can deduce the differential equation for the process:

$$(\frac{R}{\gamma_1 - 1} + \frac{R}{\gamma_2 - 1})\mathrm{d}T + p.\ \mathrm{d}V = 0$$

or

$$\frac{dT}{T} + \frac{(\gamma_1 - 1)(\gamma_2 - 2)}{\gamma_1 + \gamma_2 - 2}\frac{dV}{V} = 0 \qquad (5)$$

By putting

$$K = \frac{(\gamma_1 - 1)(\gamma_2 - 1)}{\gamma_1 + \gamma_2 - 2} = \frac{2}{11} \qquad (6)$$

after integrating (5), we have:

$$TV^K = \text{const.} \qquad (7)$$

The condensing temperature of water vapor under the pressure 0.5 atm. is also the boiling temperature T' of water under the same pressure. By using the given approximate formula, we obtain

$$\frac{1}{T'} - \frac{1}{T_0} = (-\frac{R}{\mu L})\ln\frac{p}{p_0}$$

- If we consider p approximatively constant (with relative deviation about 20/373 ≈ 5 %) T' can be easily found, and

$$T' = 354 \text{ K}$$

The volume V' of the right compartment at temperature T' can be calculated as follows:

$$V' = V_0 \left(\frac{T_1}{T'}\right)^{\frac{1}{K}} = V_0 \left(\frac{373}{354}\right)^{\frac{11}{2}} = 1.33\, V_0 \cong 1.3\, V_0$$

$$= 81.6 \text{ dm}^3 = 0.0816 \text{ m}^3 \cong 0.08 \text{ m}^3$$

b.The work done by the gas in the expansion is:

$$A = -\Delta U = (C_{V1}+C_{V2})(T_0 - T') = \left(\frac{R}{\gamma_1 - 1} + \frac{R}{\gamma_2 - 2}\right)(373 - 354) =$$

$$= \left(\frac{5}{2}R + \frac{6}{2}R\right)\times 19 = 868 \text{ J} \cong 9\times 10^2 \text{ J}$$

- If we consider the dependence of water vapor pressure p on temperature T', we must resolve the transcendental equation

$$\frac{1}{T'} - \frac{1}{T_0} = \left(-\frac{R}{\mu L}\right)\ln\frac{1}{2}\frac{T'}{T_0} = \frac{R}{\mu L}\ln 2 - \frac{R}{\mu L}\ln\frac{T'}{T_0}$$

This equation can be reduced to a numerical one:

$$\frac{1}{T'} - \frac{1}{373} = 1.422\times 10^{-4} - 2.052\times 10^{-4}\, \boldsymbol{ln}\frac{T'}{373}$$

By giving T' different values 354, 353, 352 and choosing the one which satisfies this equation, we find the approximate solution :

$$T' = 353 \text{ K}$$

With this value of temperature, the volume of the right compartment is :

$$V' = V_0\left(\frac{373}{353}\right)^{\frac{11}{2}} = 1.35\, V_0 = 0.082 \text{ m}^3 \cong 0.08 \text{ m}^3$$

b. The work done by the gas in the expansion is:

$$A = \frac{11}{2}R\times 20 = 914 \text{ J} \cong 9\times 10^2 \text{ J}$$