Physics

Competition

Theoretical Competition

Please read this first:

The time available is 5 hours for the theoretical competition.

1. Use only the pen provided.
2. Use only the one side of the paper.
3. Begin each part of the problem on a separate sheet.
4. For each question, in addition to the ***blank sheets*** where you may write, there is an ***answer sheet*** where you *must* summarize the results you have obtained. Numerical results should be written with as many digits as are appropriate to the given data.
5. Write on the blank sheets of paper whatever you consider is required for the solution of the question. Please use ***as little text as possible***; express your answers primarily in equations, numbers, figures, and plots.
6. Fill in the boxes at the top of each answer sheet of paper used by writing your student code as shown on your identification tag, and additionally on the "blank" sheets: your student code, the problem number, the progressive number of each sheet (**Page n**. from 1 to N) and the total number (N) of "blank" sheets that you use and wish to be evaluated (Page total) **for each problem**; If you use some blank sheets of paper for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
7. At the end of the competition, arrange all sheets for each problem *in the following order*:

   (a) answer sheet

   (b) used sheets in order

   (c) the sheets you do not wish to be marked

   (d) unused sheets and the printed questions

   Place the papers inside the envelope provided and leave everything on your desk.

   You are not allowed to take *any* sheets of paper out of the room.

Theoretical problem 1

Back-and-Forth Rolling of a Liquid-Filled Sphere (10 points)

Consider a sphere filled with liquid inside rolling back and forth at the bottom of a spherical bowl. That is, the sphere is periodically changing its translational and rotational direction. Due to the viscosity of the liquid inside, the movement of the sphere would be very complicated and hard to deal with. However, a simplified model presented here would be beneficial to the solution of such a problem.

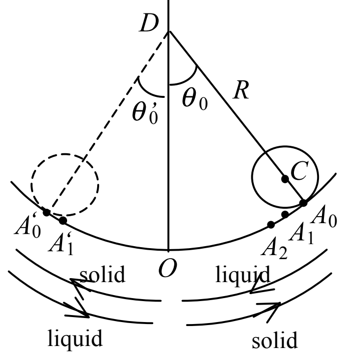
Assume that a rigid thin spherical shell of radius r and mass m is fully filled with some liquid substance of mass M, denoted as **W**. **W** has such a unique property that usually it behaves like an ideal liquid (i.e. without any viscosity), while in response to some special external influence (such as electric field) it transits to solid state immediately with the same volume; and once the applied influence removed, the liquid state recovers immediately. Besides, this influence does not give rise to any force or torque exerting on the sphere. This liquid-filled spherical shell (for convenience, called 'the sphere' hereafter) is supposed to roll back and forth at the bottom of a spherical bowl of radius R ($R > r$) without any relative slipping, as shown in the figure. Assume the sphere moves only in the vertical plane (namely, the plane of the figure), please study the movement of the sphere for the following three cases:

1. **W** behaves as in ideal solid state, meanwhile **W** contacts the inner wall of the spherical shell so closely that they can be taken as solid sphere as a whole of radius r with an abrupt density change across the interface between the inside wall of the shell and **W**.

   (1) Calculate the rotational inertia I of the sphere with respect to the axis passing through its center C. (You are asked to show detailed steps.)

   (1.0 points)

   (2) Calculate the period T_1 of the sphere rolling back and forth with a small amplitude without slipping at the bottom of the spherical bowl. (2.5points)



2. **W** behaves as an ideal liquid with no friction between **W** and the spherical shell. Calculate the period T_2 of the sphere rolling back and forth with a small amplitude without slipping at the bottom of the spherical bowl. (2.5 points)

3. **W** transits between ideal solid state and ideal liquid state.

   Assume at time $t=0$, the sphere is kept at rest, the line *CD* makes an angle θ_0 ($\theta_0 << 1\text{rad}$) with the plumb line *OD*, where *D* is the center of the spherical bowl. The sphere contacts the inner wall of the bowl at point A_0, as shown in the figure. Release the sphere, it starts to roll left from rest. During the motion of the sphere from A_0 to its equilibrium position *O*, **W** behaves as ideal liquid. At the moment that the sphere passes through point *O*, **W** changes suddenly into solid state and sticks itself firmly on the inside wall of the sphere shell until the sphere reaches the left highest position A_0'. Once the sphere reaches A_0' , **W** changes suddenly back into the liquid state. Then, the sphere rolls right; and **W** changes suddenly into solid state and sticks itself firmly on the inside wall of the spherical shell again when the sphere passes through the equilibrium position *O*. When the sphere reaches the right highest position A_1, **W** changes into liquid state once again. Then the whole circle repeats time after time. The sphere rolls right and left periodically but with the angular amplitude decreased time after time. The motion direction of the sphere is shown by curved arrows in the figure, together with the words "solid" and "liquid" showing corresponding state of **W**. It is assumed that during such process of rolling back and forth, no any relative slide happens between the sphere and the inside wall of the bowl (or, alternatively, the bottom of the bowl can supply as enough friction as needed). Calculate the period T_3 of the sphere rolling right and left, and the angular amplitude θ_n of the center of the sphere, namely, the angle that the line *CD* makes with the vertical line *OD* when the sphere reaches the right highest position A_n for the n-th time (only A_2 is shown in the figure). （4.0 points）