
Problem 3. How does a superluminal object look like?

1. Radiating superluminal dot

(1) Expression of t in terms of d, t', u and v.

$t = t' + \sqrt{d^2 + (vt')^2}/u$	1.0

(2) The apparent position x'_0 in terms of d and θ.

$x'_0 = -d\cot\theta$	1.0

The observed time t_0 of the first appearance in terms of d, v and ϑ.

$t_0 = \frac{d}{v}\tan\theta$	1.0

(3) The apparent position(s) $x'(t)$ in terms of v, θ, t and t_0.

$x'_+ = v\cot^2\theta\left(-t + \cos^{-1}\theta\sqrt{t^2 - {t_0}^2}\right)$ $x'_- = v\cot^2\theta\left(-t - \cos^{-1}\theta\sqrt{t^2 - {t_0}^2}\right)$	2.0

(4) The apparent velocity(s) $v'(t)$ in terms of v, θ, t and t_0.

$v'_+(t) = v\cot^2\theta\{-1 + 1/[\cos\theta\sqrt{1-(t_0/t)^2}]\}$ along the $+x$ axis $v'_-(t) = v\cot^2\theta\{-1 - 1/[\cos\theta\sqrt{1-(t_0/t)^2}]\}$ along the $-x$ axis	1.0

(5) The apparent velocity(s) v' of the first appearance of the particle

$v'_+ = -\infty$ along the $+x$ axis $v'_- = \infty$ along the $-x$ axis	0.2

(6) The apparent velocity(s) v' of the particle at infinite distances in terms of v and u

$v'_+ = vu/(v+u) = u/(1+\cos\theta)$ along the $+x$ axis $v'_- = -vu/(v-u) = -u/(1-\cos\theta)$ along the $-x$ axis	0.2

(7) The graph of the apparent velocity v' versus time t. (Remember to write down the asymptotic values of the apparent velocity).

	1.0

(8) An apparent velocity CAN / CANNOT exceed the light speed in the vacuum. Circle the correct answer.

Can	0.2
Can not	

1.1. Radiating linear object

A. Parallel movement

(9) The time interval of complete appearance of the whole linear object from the first appearance of its front point. (in terms of L, γ and v)

$\Delta t = L/(\gamma v)$	0.3

(10) The apparent length(s) of the object at the moment of its complete appearance. (in terms of d , L, θ and γ)

$L_+ = \frac{L\cot^2\theta}{\gamma}\left(\cos^{-1}\theta\sqrt{1+\frac{2d\gamma\tan\theta}{L}}-1\right)$ $L_- = \frac{L\cot^2\theta}{\gamma}\left(\cos^{-1}\theta\sqrt{1+\frac{2d\gamma\tan\theta}{L}}+1\right)$	0.4

B. Perpendicular movement

(11) Show that the x and y coordinates of any given point of the object satisfy an elliptic equation

$\frac{(x-x_c)^2}{a^2}+\frac{y^2}{b^2}=1$	0.7

(12) The position x_c of the centre of symmetry of the ellipse in terms of v, t and θ.

$x_c = -vt\cot^2\theta$	0.5

(13) The lengths of the semi-major and semi-minor axes of the ellipse in terms of v, t and θ.

$a = vt\frac{\cos\theta}{\sin^2\theta}$ $b = vt\cot\theta$	0.5