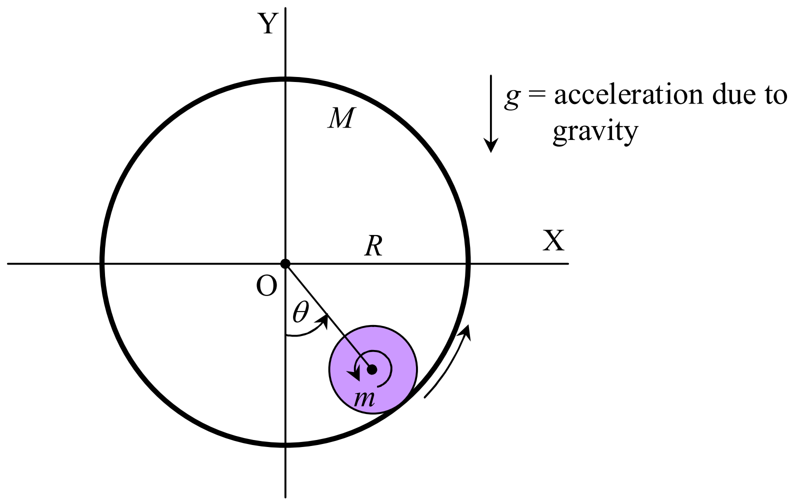
Rolling Cylinders

A thin-walled cylinder of mass M and rough inner surface of radius R can rotate about its fixed central horizontal axis OZ. The Z-axis is perpendicular to and out of the page. Another smaller uniform solid cylinder of mass m and radius r rolls without slipping (except for question 1.8) on the inner surface of M about its own central axis which is parallel to OZ.



1.1) The rotation of M is to be started from rest at the instant $t = 0$ when m is resting at the lowest point. At a later time t the angular position of the centre of mass of m is θ and by then M has turned through an angle ϕ radians. How many radians (designated ψ) would have mass m turned through about its central axis relative to a fixed line (for example, the negative Y-axis)? Give your answer in terms of θ, ϕ, R and r. (0.8 point)

1.2) What is the angular acceleration of m, $\frac{d^2}{dt^2}\psi$, about its own axis through its centre of mass? Give your answer in terms of $R, r,$ and derivatives of θ and ϕ. (0.2 point)

1.3) Derive an equation for the angular acceleration of the centre of mass of m, $\frac{d^2}{dt^2}\theta$, in terms of $m, g, R, r, \theta, \frac{d^2}{dt^2}\phi$, and the moment of inertia I_{CM} of m about its central axis. (1.8 points)

1.4) What is the period of small amplitude oscillation of m when M is constrained to rotate at a constant angular velocity? Give your answer only in terms of $R, r,$ and g. (1.3 point)

1.5) What is the value of θ for the equilibrium position of m in question 1.4? (0.2 point)

1.6) What is the equilibrium position of m when M is rotating with a constant angular acceleration α? Give your answer in terms of $R, g,$ and α. (0.7 point)

1.7) Now M is allowed to rotate (oscillate) freely, without constraint, about its central axis OZ while m is executing a small-amplitude oscillation by pure rolling on the inner surface of M. Find the period of this oscillation. (2.5 points)

1.8) Consider the situation in which M is rotating steadily at an angular velocity Ω and m is rotating (rolling) about its stationary centre of mass, at the equilibrium position found in question 1.5. M is then brought abruptly to a halt. What must be the lowest value of Ω such that m will roll up and reach the highest point of the cylindrical surface of M? The coefficient of friction between m and M is assumed to be sufficiently high that m begins to roll without slipping soon after a short skidding right after M is stopped. (2.5 points)

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