



equilibrium positions at finite temperature and participate in heat conduction process. If the resistance of the thermocouple is neglected, these two subsystems may be considered as noninteracting, the electron gas exchanges heat only with the heat source at  $T_1$  and the heat sink at  $T_2$ , performing the ideal Carnot cycle.

<b>A2.1</b> <b>0.25 pt</b>	Electron gas receives heat from heat source due to the Peltier effect $q_1 = \pi_1 I$ (A10)	<b>0.25</b>
<b>A2.2.</b> <b>0.25 pt</b>	The heat amount transferred to the heat sink due to the Peltier effect $q_2 = \pi_2 I$ (A11)	<b>0.25</b>
<b>A2.3.</b> <b>0.5 pt</b>	Power delivered by the electron gas due to the Seebeck emf is $P = \varepsilon I = \alpha (T_1 - T_2) I$ (A12)	<b>0.5</b>
<b>A2.4</b> <b>0.5 pt</b>	The efficiency of the ideal Carnot cycle applied to the thermocouple can be written as $\eta = \frac{P}{q_1}, \eta = \frac{T_1 - T_2}{T_1}$ . (A13) Thus $\frac{T_1 - T_2}{T_1} = \frac{\alpha (T_1 - T_2)}{\pi_1}$ (A14)	<b>0.25</b>  <b>0.25</b>
	Comparing these equations, one has $\pi_1 = \alpha T_1$ . This is the Peltier coefficient at the first junction contacting with the heat source. Generally, one has $\pi = \alpha T$ .	

### A3. Thermoelectric generator

<b>A3.1.</b> <b>0.5 pt</b>	Power received by the thermocouple from the heat source (see also (A8)) is $q_1 = K(T_1 - T_2) + \alpha T_1 I - \frac{1}{2} I^2 R$ . (A15) Here $\alpha$ is the Seebeck coefficient of the thermocouple and $K = K_A + K_B = \frac{k_A S_A}{L} + \frac{k_B S_B}{L}$ , (A16) $R = R_A + R_B = \frac{\rho_A L}{S_A} + \frac{\rho_B L}{S_B}$ , (A17) are its thermal conductance and internal resistance. The heat sink receives a power (see also (A9)) $q_2 = K(T_1 - T_2) + \alpha T_2 I + \frac{1}{2} I^2 R$ . (A.18)	<b>0.25</b>     <b>0.25</b>
<b>A3.2.</b> <b>0.75 pt</b>	The efficiency of the thermoelectric generator is $\eta = \frac{P_L}{q_1} = \frac{I^2 R_L}{K(T_1 - T_2) + \alpha T_1 I - I^2 R / 2} = \frac{m}{\frac{K(T_1 - T_2)}{I^2 R} + \frac{\alpha T_1}{IR} - \frac{1}{2}}$ . (A19) Here we use $R_L = mR$ . The electrical current in the circuit is	<b>0.25</b>

	$I = \frac{\alpha(T_1 - T_2)}{R_L + R} = \frac{\alpha(T_1 - T_2)}{(1+m)R}. \quad (\text{A20})$ <p>Substituting (A20) into (A19) we obtain the expression for the efficiency</p> $\eta = \frac{m(T_1 - T_2)}{\frac{KR(1+m)^2}{\alpha^2} + T_1(1+m) - \frac{T_1 - T_2}{2}}. \quad (\text{A21})$	<b>0.25</b>
<b>A3.3.</b> <b>0.25</b>	<p>Replacing the figure of merit</p> $Z = \frac{\alpha^2}{KR} \quad (\text{A22})$ <p>and <math>\eta_c = \frac{T_1 - T_2}{T_1}</math> the efficiency of the ideal Carnot cycle in (A21), one has</p> $\eta = \eta_c \frac{m}{\frac{(1+m)^2}{ZT_1} + (1+m) - \frac{1}{2}\eta_c}. \quad (\text{A23})$ <p>From (A23) one sees that larger <math>Z</math> leads to the larger efficiency of the corresponding thermoelectric generator. The condition <math>ZT_1 \geq 1</math> can be used for material application in thermoelectric generators.</p>	<b>0.25</b>

#### A4. The maximum efficiency

<b>A4.1</b> <b>0.25 pt</b>	<p>When <math>R_L = R</math> or <math>m=1</math>, the power consumed on the load is maximum. The efficiency in that case is</p> $\eta_P = \frac{T_1 - T_2}{\left[ \frac{4}{Z} + \frac{3T_1 + T_2}{2} \right]}. \quad (\text{A24})$	<b>0.25</b>
<b>A4.2.</b> <b>0.75 pt</b>	<p>Equation (A23) may be rewritten as</p> $\eta = \frac{m}{a(1+m)^2 + b(1+m) - 1/2}, \quad (\text{A25})$ <p>where <math>a = \frac{1}{Z(T_1 - T_2)}</math>, <math>b = \frac{T_1}{T_1 - T_2}</math>.</p> <p>Equation <math>\frac{d\eta}{dm} = 0</math> has the solution <math>M = \sqrt{1 + \frac{2b-1}{2a}}</math> or</p> $M = \sqrt{1 + Z \frac{(T_1 + T_2)}{2}}. \quad (\text{A26})$	<b>0.25</b>
<b>A4.3.</b> <b>0.25 pt</b>	<p>Using (A25), (A26) we obtain the maximum efficiency of the thermoelectric generator</p> $\eta_{\max} = \frac{T_1 - T_2}{T_1} \frac{(M-1)}{\left( M + \frac{T_2}{T_1} \right)} \quad (\text{A27})$ <p>(Correct expression containing either <math>M</math>, <math>Z</math> or both is also accepted)</p>	<b>0.25</b>

### A5. The maximum figure of merit

<p><b>A5.1</b> <b>0.5</b></p>	<p>According to (A22) <math>Z</math> takes the maximum value <math>Z = Z_m</math> when <math>KR = y</math> is smallest. Denoting <math>(k_A S_A + k_B S_B) \left( \frac{\rho_A}{S_A} + \frac{\rho_B}{S_B} \right) = y</math>, <math>x = \frac{S_A}{S_B}</math> one has the equation <math>(k_A x + k_B) \left( \frac{\rho_A}{x} + \rho_B \right) = y</math>.</p> <p>It is easily to show the function <math>y</math> has the minimum at <math>x = x_m</math>, where</p> $x_m = \sqrt{\frac{\rho_A k_B}{\rho_B k_A}} \quad \text{or} \quad \frac{S_A}{S_B} = \left( \frac{\rho_A k_B}{\rho_B k_A} \right)^{1/2}. \quad (\text{A28})$	<p><b>0.25</b></p> <p><b>0.25</b></p>
<p><b>A5.2</b> <b>0.25 pt</b></p>	<p>If the ratio of cross-sectional areas satisfies (A28) then</p> $y_m = \left[ (\rho_A k_A)^{1/2} + (\rho_B k_B)^{1/2} \right]^2$ <p>and the maximum figure of merit of the thermocouple is</p> $Z_m = \frac{\alpha^2}{\left[ (\rho_A k_A)^{1/2} + (\rho_B k_B)^{1/2} \right]^2}. \quad (\text{A.29})$	<p><b>0.25</b></p>

### A6. The optimal efficiency

<p><b>A6.1.</b> <b>0.5 pt</b></p>	<p>The thermocouple with two bars made from material A and B has the following the figure of merit</p> $Z_m = \frac{\alpha^2}{\left[ (\rho_A k_A)^{1/2} + (\rho_B k_B)^{1/2} \right]^2} = \frac{\alpha^2}{4\rho_A k_A} = 3.15 \times 10^{-3} \text{ K}^{-1}. \quad (\text{A.30})$ <p>The optimal efficiency of the thermocouple AB when <math>T_1 = 423\text{K}</math>, <math>T_2 = 303\text{K}</math> has the following value</p> $\eta_{opt} = \frac{T_1 - T_2}{4Z_m^{-1} + \frac{3T_1 + T_2}{2}} = \frac{120}{4 \frac{1}{3.2 \times 10^{-3}} + \frac{3 \times 423 + 303}{2}} = 5.84\%. \quad (\text{A.31})$ <p>The corresponding ideal Carnot efficiency for that case is</p> $\eta_C = \frac{T_1 - T_2}{T_1} = \frac{120}{423} = 28.4\% \quad (\text{A32})$ <p><math>\eta_{opt} / \eta_C = 0.21</math>.</p>	<p><b>0.15</b></p> <p><b>0.25</b></p> <p><b>0.1</b></p>
<p><b>A6.2</b> <b>0.25 pt</b></p>	<p>The maximum efficiency of the thermoelectric generator designed from AB materials is</p> $M = \sqrt{1 + Z_m \frac{(T_1 + T_2)}{2}} = \sqrt{1 + 3.2 \times 10^{-3} \times 363} = 1.46$ $\eta_{max} = \eta_C \frac{(M - 1)}{\left( M + \frac{T_2}{T_1} \right)} = 6.0\% \quad (\text{A.33})$	<p><b>0.25</b></p>



### B3. The coefficient of performance

<p><b>B3.1</b> <b>0.5pt</b></p>	<p>According to the energy conservation law, the power supplied by the electrical source <math>P</math> equals to the Joule heat plus Peltier's heat taken away in thermocouple per unit of time:</p> $P = \alpha(T_2 - T_1)I + RI^2 \quad (B.8)$ <p>The equation for Coefficient of Performance (COP) is</p> $\beta = \frac{q_c}{P} = \frac{\alpha T_1 I - K(T_2 - T_1) - \frac{RI^2}{2}}{\alpha(T_2 - T_1)I + RI^2} \quad (B.9)$	<p><b>0.25</b></p> <p><b>0.25</b></p>
<p><b>B3.2.</b> <b>0.25</b></p>	<p>Electrical current <math>I_\beta</math> corresponds to the maximum of the COP is found from the equation <math>\frac{d\beta}{dI} = 0</math>. (B.9) may be rewritten in convenience form</p> $\beta = -\frac{1}{2} + \frac{\alpha(T_1 + T_2)I - 2K(T_2 - T_1)}{2[\alpha(T_2 - T_1) + RI]I} \quad (B.10)$ <p>The equation <math>\frac{d\beta}{dI} = 0</math> leads to</p> $-\alpha R(T_1 + T_2)I^2 + 4K(T_2 - T_1)RI + 2K\alpha(T_2 - T_1)^2 = 0,$ $I^2 - \frac{2K(T_2 - T_1)I}{\alpha T_M} - \frac{K}{RT_M}(T_2 - T_1)^2 = 0, \quad (B.11)$ <p>with <math>T_M = \frac{(T_2 + T_1)}{2}</math>. <span style="float: right;">(B.12)</span></p> <p>Solution of (B.11) is</p> $I_\beta = \frac{K(T_2 - T_1)}{\alpha T_M} \left\{ \sqrt{1 + Z T_M} + 1 \right\}. \quad (B.13)$ <p>(Taking into account that <math>Z = \frac{\alpha^2}{KR}</math>, (B.13) can be written in other form</p> $I_\beta = \frac{\alpha(T_2 - T_1)}{R \left\{ \sqrt{1 + Z T_M} - 1 \right\}} \quad (B.14)$	<p><b>0.25</b></p>
<p><b>B3.3.</b> <b>0.25</b></p>	<p>Substituting (B.14) into (B.9) one has</p> $\beta_{\max} = \frac{T_1 \left[ \sqrt{1 + Z T_M} - T_2 / T_1 \right]}{(T_2 - T_1) \left[ \sqrt{1 + Z T_M} + 1 \right]}. \quad (B.15)$	<p><b>0.25</b></p>