Water-Hammer Effect

Introduction

This problem studies variations of fluid pressure caused by pressure waves in a flow pipe. Proposed tasks mainly deal with the water-hammer effect arising from both fast and slow closings of a flow-control valve in the pipe.

We consider only nonviscous liquids and liquid flows which are essentially one-dimensional. All pipes including their valves are assumed to be rigid, but liquids are not always considered to be incompressible. If a liquid element of volume V_0 at equilibrium under pressure P_0 is subjected to a change of pressure ΔP , the change of its volume ΔV is assumed to be proportional to ΔP so that

$$\Delta P = -B\,\frac{\Delta V}{V_0} \tag{1}$$

The constant of proportionality$B$ represents the bulk modulus of the liquid. For water, take $\rho_0 = 1.0 \times 10^3$ kg/m^3 as its equilibrium density and $B = 2.2$ GPa.

Part A. Excess Pressure and Propagation of Pressure Wave (2.2 points)

In a uniform cylindrical pipe of length L, water is flowing steadily along the $+x$ direction with horizontal velocity v_0, density ρ_0, and pressure P_0. As shown in Fig. 1, the pipe is connected to a reservoir at a depth h and opens into the atmosphere at pressure P_a.

Suppose the flow-control valve T at the end of the pipe is then shut instantly so that the oncoming liquid element next to the valve suffers both a pressure change $\Delta P_s \equiv P_1 - P_0$ and a velocity change $\Delta v = v_1 - v_0$ with $v_1 \le 0$. This causes a longitudinal wave of excess pressure ΔP_s to travel upstream in the $-x$ direction with a speed of propagation c.

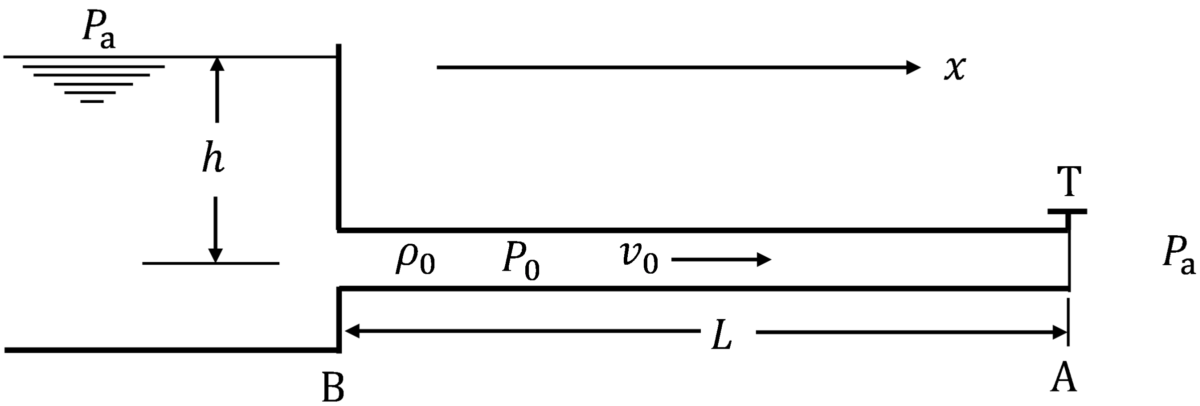

Fig. 1: Steady flow in a uniform pipe.

A.1	The excess pressure ΔP_s is related to the velocity change Δv by $\Delta P_s = \alpha \rho_0 c \Delta v$. The speed of propagation c is given by $c = \beta + \sqrt{\gamma B/\rho_0}$. Find α, β, and γ.	1.6pt

A.2	Calculate values of c and ΔP_s for the case of water flow with $v_0 = 4.0$ m/s and $v_1 = 0$.	0.6pt

Part B. A Model for the Flow-control Valve (1.0 points)

Fig. 2 shows a model for control valve T and the liquid flow through it. The valve is taken to be a short section of length ΔL and inner radius R near the end A of the pipe. Its cone-shaped outlet has an orifice of radius r and opens into the atmosphere at pressure P_a. Effects of gravity on the efflux are to be neglected.

The liquid is to be regarded as incompressible and the flow as steady with liquid element at the valve inlet having velocity v_{in}, pressure P_{in}, and density ρ_0. In Fig. 2, stream lines and normal lines are drawn only as an aid for visualizing the flow pattern.

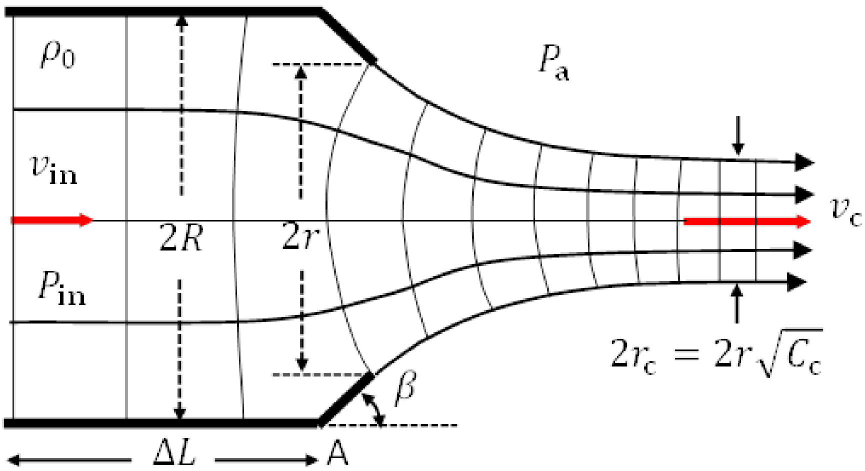

Fig. 2: Valve dimensions and contraction of jet.

It is known that, after leaving the valve into the atmosphere, the cross section of the flow will contract until it reaches a minimum where the stream lines are again parallel. At this point of minimum, the flow velocity is v_c and the cross section of the flow has a radius $r_c = r\sqrt{C_c}$. Here C_c, called the **contraction coefficient**, depends on the ratio r/R and the cone angle β as shown in Table 1.

r/R	$C_c(\beta = 45^\circ)$	$C_c(\beta = 90^\circ)$
0.00	0.746	0.611
0.20	0.747	0.616
0.30	0.748	0.622
0.40	0.749	0.631
1.00	1.000	1.000

Table 1. Contraction Coefficients for Orifices

B.1	Find the excess pressure $\Delta P_{\text{in}} = P_{\text{in}} - P_{\text{a}}$ at the valve inlet where the stream lines are parallel. Give your answer in terms of ρ_0, v_{in}, r, R, and C_{c}.	1.0pt

For all tasks in Part C and Part D, we consider the reservoir-pipe system in Fig. 1 and make the following assumptions:

- Speed of propagation c and density ρ_0 of liquid are given constants independent of flow velocity. The ambient atmospheric pressure P_{a} and the acceleration of gravity g are constant.
- Initially, the valve is fully open and the flow in the pipe is steady with fluid pressure P_0 and velocity v_0.
- As in Fig. 1 and Fig. 2, the pipe has length L and radius R. The valve T is a circular opening of variable radius r with angle $\beta = 90^\circ$ and its length ΔL is negligible so that the valve inlet is effectively at the end A of the pipe. Effects of gravity on the efflux are negligible.
- Liquid in the reservoir is quasi-static so that its pressure P_h near the pipe entrance B remains constant and we assume that the variation of fluid pressure across the pipe is negligible so that the flow is one-dimensional throughout the pipe.
- The model outlined in Part B may be used to determine the excess pressure $\Delta P_{\text{in}} = P_{\text{in}} - P_{\text{a}}$ at the valve inlet.

Part C. Water-Hammer Effect due to Fast Closure of Flow Control Valve (1.8 points)

Refer to the reservoir-pipe system in Fig. 1. When liquid flow in the pipe is obstructed by complete or partial closure of the valve, a pressure wave starts traveling upstream. It gets reflected at the reservoir end of the pipe and travels back to the valve and gets reflected there. Then another pressure wave is generated and the process just described is repeated. This causes a sequence of sudden pressure surges and dips for liquid element next to the valve and is referred to as **water-hammering**.

C.1	Refer to Fig. 1 and Fig. 2. Find the pressure P_0 and velocity v_0 of the steady flow in the pipe when valve T is fully open ($r = R$). Give answers in terms of ρ_0, g, h, and P_{a}.	0.6pt

C.2	Consider the same steady flow as in task C.1 with pressure P_0 and flow velocity v_0. Now, at $t = 0$, the valve is closed ($r = 0$) instantly. A pressure wave heads toward the reservoir with speed of propagation c. Take note $P_h = P_0 + \rho_0 g h$. Let $\tau = 2L/c$. What are the fluid pressure $P(t)$ and flow velocity $v(t)$ in the pipe when t is getting very close to each of the instants $\tau/2$ and τ?	1.2pt

Part D. Water-Hammer Effect due to Slow Closure of Flow Control Valve (5.0 points)

Consider again the same steady flow as in task C.1 with pressure P_0 and flow velocity v_0. Now we close the valve slowly and adopt a finite-step approach to simulate the closing process.

Starting at time $t = 0$, the instant reduction of the radius r of the valve (see Fig. 2) is carried out sequentially at a time interval $\tau = 2L/c$. Immediately after each instant reduction of radius, the flow in the valve region is approximated to be steady as in Part B. The pressure and velocity at the valve are then different from those of the rest of the flow in the pipe.

For each closing step n, its duration and the radius r_n of the valve opening are specified in Table 2 along with the symbols used to represent the corresponding fluid pressure P_n and flow velocity v_n at the valve.

closing step n	time interval of step n	ratio r_n/R	pressure at valve when $t=(n-1)\tau$	flow velocity at valve when $t=(n-1)\tau$
$n=0$	$t<0$	1.00	P_0	v_0
$n=1$	$0\le t<\tau$	0.40	P_1	v_1
$n=2$	$\tau\le t<2\tau$	0.30	P_2	v_2
$n=3$	$2\tau\le t<3\tau$	0.20	P_3	v_3
$n=4$	$3\tau\le t<4\tau$	0.00	P_4	$v_4=0$

Table 2. Valve closing steps

Take fluid density ρ_0 and speed of propagation c as constants. Let $n=0,1,2,3,4$. Define $\Delta P_n=P_n-P_0$ and $\Delta v_n=v_n-v_0$. Make sure to enforce the approximation $P_h=P_0$.

D.1 Obtain an equation which expresses $\Delta P_n/(\rho_0 c)$ in terms of $\Delta P_{n-1}/(\rho_0 c)$, v_{n-1}, and v_n. It must be valid for all steps $n>0$ specified in Table 2. For $n=1,2,3$, obtain also an equation which allows v_n to be computed if both v_{n-1} and $\Delta P_{n-1}/(\rho_0 c)$ are known. 3.0pt

D.2 Apply the result of task D.1 to water flow with $v_0=4.0$ m/s. Use the graph paper provided in the Answer Sheet to make all plots of ΔP versus $\rho_0 c v$. Be sure to draw lines and curves intersecting at points having coordinates which give the values of $\rho_0 c v_n$ and ΔP_n for steps $n=1,2,3,4$. On the plot, label each point of intersection $(\rho_0 c v_n, \Delta P_n)$ with the value of n to which it corresponds. From the graph, estimate values of $\rho_0 c v_n$ and ΔP_n (both in units of MPa) for $n=1,2,3,4$. 2.0pt